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THE NECESSARY CONDITIONS OF THE SLOW CONVERGENCE FOR A CLASS OF SOLUTION METHODS FOR THE INVERSE CAUCHY PROBLEM IN BANACH SPACE

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1. Consider the problem of approximate solution of the ill-posed operator equation

$$Bu = f, \quad u \in X, \tag{1}$$

where the linear bounded operator B which acts in a Banach space X is not assumed to be continuously invertible. Many works are devoted to solution methods for such problems (see, e. g., [1]–[4] and references therein). Assume that problem (1) has a unique solution u^*. We approximate the solution by one method within the general scheme ([4], p. 39)

$$u_\alpha = (E - \Theta(B,\alpha)B)\xi + \Theta(B,\alpha)f, \quad \alpha \in (0, \alpha_0], \tag{2}$$

which defines a family of elements $u_\alpha \in X$ such that $\lim\limits_{\alpha\to 0} u_\alpha = u^*$. In formula (2) we denote by ξ the initial approximation of the solution and understand the function of the operator $\Theta(B,\alpha)$ in terms of the appropriate operator calculus. Under the corresponding choice of the generating function $\Theta(\cdot,\alpha)$, scheme (2) allows one to obtain the M.M. Lavrentiev method and its iterated version, the method of asymptotic regularization, the simplest explicit and implicit iterative methods. Since problem (1) is ill-posed, the convergence of approximations u_α to the solution u^* can be arbitrarily slow, if the solution is not subjected to additional requirements, for example, to conditions of sourcewise representability. Thus, if the operator B satisfies the sectorial condition and the initial residual admits the sourcewise representation

$$u^* - \xi \in R(B^p), \quad p > 0, \tag{3}$$

then the convergence rate meets the power estimate ([4], p. 42)

$$\|u_\alpha - u^*\| \le C_1\alpha^p, \quad \alpha \in (0, \alpha_0]. \tag{4}$$

Hereinafter $C_1, C_2, \dots$ are positive constants. Condition (3) which is sufficient for estimate (4) is close to the necessary one. Namely, formula (4) implies the representation ([4], p. 63)

$$u^* - \xi \in R(B^q) \quad \forall q \in (0,p).$$

The same assertions are valid, if B is a self-conjugate nonnegative operator, acting in the Hilbert space (e. g., [2], [3]). Further, for $B = B^* \ge 0$ in the Hilbert space the representation

$$u^* - \xi \in R((-\ln B)^{-p}), \quad p > 0, \tag{5}$$

implies the estimate

$$\|u_\alpha - u^*\| \le C_2(-\ln\alpha)^{-p}, \quad p > 0, \quad \alpha \in (0, \alpha_0], \tag{6}$$