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SOLUTION OF A TWO-DIMENSIONAL WEAKLY SINGULAR INTEGRAL EQUATION OF FIRST GENUS

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This article is devoted to both exact and approximate methods for solving two-dimensional weakly singular integral equation (w.s.i.e.) of first genus with logarithmic kernels of the form

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\ln\left|\sin\frac{s-\sigma}{2}\right|\ln\left|\sin\frac{t-\tau}{2}\right|x(\sigma,\tau)d\sigma\,d\tau + \\ + \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}h(s-\sigma,t-\tau)x(\sigma,\tau)d\sigma\,d\tau = y(s,t); \quad (1)$$

here $h(s,t)$, $y(s,t)$ are known continuous 2π-periodic functions with respect to each of variables, $x(\sigma,\tau)$ is the sought-for function, and the singular integral is treated as the improper one.

As in the one-dimensional case (see [1]–[3]), the problem of solving equation (1) is ill-posed. However, a specific structure of the kernel allows us to pose this problem in a correct way in a pair of functional spaces which are defined as follows. In the capacity of the space of the desired elements X we shall consider the space $L_2 \equiv L_2[0,2\pi]^2$ of square-integrable 2π-periodic functions of two variables, with the ordinary norm

$$\|x\|_{L_2} = \left(\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}|x(\sigma,\tau)|^2 d\sigma\,d\tau\right)^{1/2}, \quad x \in L_2.$$

Next, in the capacity of the space of right sides, we use the space $Y = W_2^{12} \equiv W_2^{12}[0,2\pi]^2$ of 2π-periodic functions $y(s,t)$ of two variables, which along with their first order y'_1, y'_2 and second order mixed derivatives $y''_{12} = y''_{21}$ are square-integrable 2π-periodic functions of two variables. In the space W_2^{12} we introduce the norm

$$\|y\|_{W_2^{12}} = 4[\|y\|_X + \|y'_1\|_X + \|y'_2\|_X + \|y''_{12}\|_X].$$

Everywhere in what follows we assume that the w.s.i.e. (1) is uniquely resolvable in the space $X = L_2 \equiv L_2[0,2\pi]^2$ for any right side $y(s,t) \in Y = W_2^{12} \equiv W_2^{12}[0,2\pi]^2$.

We shall consider the initial equation (1) in the operator form

$$Ax \equiv Sx + Rx = y \quad (x \in X, \quad y \in Y),$$

where

$$Sx \equiv \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\ln\left|\sin\frac{s-\sigma}{2}\right|\ln\left|\sin\frac{t-\tau}{2}\right|x(\sigma,\tau)d\sigma\,d\tau;$$

$$Rx \equiv \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}h(s-\sigma,t-\tau)x(\sigma,\tau)d\sigma\,d\tau.$$

The known results cited in [1] yield the following two lemmas.

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