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On Structure Equations of a Three-Web Defined by the Core of a Given Left Bol Three-Web

G. A. Tolstikhina[†] **and A. M. Shelekhov**[*]

Tver State University, ul. Zhelyabova 33, Tver, 170100 Russia

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Abstract—We give a solution to the problem stated by M. A. Akivis in 1976: Find the structure equations of the 3-web defined by the core of a given Bol 3-web.

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This paper continues the authors' study of multidimensional Bol three-webs given on a $2r$-dimensional differentiable manifold $\mathcal{M}$ by three codimension r smooth foliations λ_1, λ_2, and λ_3. Bol webs (left B_l, right B_r, and middle B_m) are characterized by the fact that all sufficiently small corresponding Bol configurations are closed ([5], P. 46).

On the figure, the left Bol configuration is shown. This figure is obtained as follows. Let a and b be two arbitrary sufficiently close vertical leaves of the web, y an arbitrary horizontal leaf (on Figure, the leaves of the first, the second, and the third foliations of the web are represented by vertical, horizontal, and slant lines, respectively). The leaf y meets the leaf a at A. The slant leaf passing through A meets the leaf b at B. The horizontal leaf passing through B meets the vertical leaf a at C; the slant leaf passing through C meets the leaf y at D. The similar construction for another horizontal leaf $\overline{y}$ leads to the points $\overline{A}$, $\overline{B}$, $\overline{C}$, and $\overline{D}$. For an arbitrary web, the vertical leaf passing through $\overline{D}$ does not pass through D. If it passes through D, one says that the left Bol configuration closes on the web.

The fact that all sufficiently small left Bol configurations are closed means that the position of the leaf c does not depend on the choice of a horizontal leaf y and depends only on the leaves a and b. Consequently, on the first (vertical) foliation of a left Bol web, there arises a binary operation $c = a * b$, which is a smooth local quasigroup called the core of the left Bol web. The core $c = a * b$ of the three-web B_l defines on the base X of the first foliation of the web a family of smooth functions S_a such that

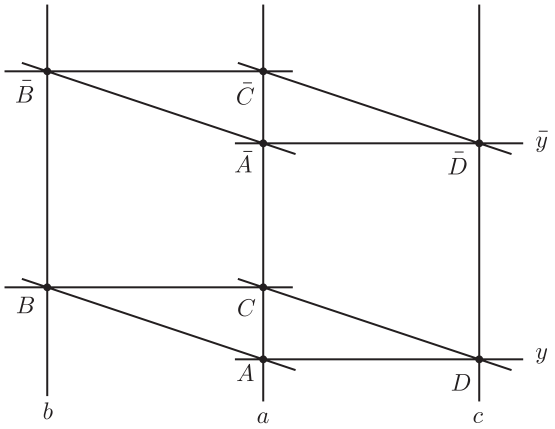

Figure.

[†]Deceased.

[*]E-mail: amshelekhov@rambler.ru.