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The Method of Mechanical Quadratures for Integral Equations with Fixed Singularity

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Abstract—We investigate the method of mechanical quadratures for integral equations with fixed singularity. We establish estimates of the error of this method based on a quadrature process, which is the best in the class of differentiable functions. We prove the convergence of the method in finite-dimensional and uniform metrics. We find that the investigated quadrature method is optimal by order on the Hölder class of functions.

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1. THE MAIN CONCEPTS AND AUXILIARY PROPOSITIONS

The method of mechanical quadratures is one of important and useful methods for solving of integral equations. A great body of publications is dealing with this method (see, e.g., [1–4]). Let us note that the method is applicable for singular integral equations, too [4, 5]. N. S. Gabbasov and his disciples obtained important results on the direct methods for solving integral equations of the second and third kinds, including the equations with fixed singularities, during the last fifteen years; see [6–9].

The present paper continues studies on the integral equations. It deals with the proof of convergence of the method of mechanical quadratures for singular integral equations with fixed singularity.

We consider integral equation

$$u(x) - \int_0^1 \frac{K(x,t)}{t} u(t)dt = f(x), \quad 0 \le x \le 1, \tag{1}$$

where $K(x,t)$ and $f(x)$ are given functions.

We denote by K operator determined by the integral term of Eq. (1):

$$Ku \equiv \int_0^1 \frac{K(x,t)}{t} u(t)dt.$$

Then we rewrite Eq. (1) in operator form

$$Au = f, \tag{2}$$

where $A = E - K$, and E is unit operator. We assume that $K(x,t)$ satisfies the following restrictions ($\mathbf{K}$):

1) $K(x,0) = 0$;

2) $|K(x,t_1) - K(x,t_2)| \le M_1|t_1 - t_2|^\alpha$, $0 < \alpha \le 1$, $M_1 = \mathrm{const} > 0$;

3) $|K(x_1,t) - K(x_2,t)| \le M_2 t^\alpha |x_1 - x_2|^\alpha$, $0 < \alpha \le 1$, $M_2 = \mathrm{const} > 0$.

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