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Initial-Boundary Problem for Parabolic-Hyperbolic Equation with Loaded Summands

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Abstract—In this paper we find necessary and sufficient conditions for uniqueness of solution to the initial-boundary problem for a loaded equation of mixed parabolic-hyperbolic type. The solution is constructed as a sum of a series in eigenfunctions of the corresponding one-dimensional problem on eigenvalues. In the proof of convergence of the series the problem of small denominators arises. Under certain conditions on these problems we obtain an estimate for a small denominator to be separated from zero that allows to prove the existence theorem in the class of regular solutions.

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1. INTRODUCTION

Consider a loaded mixed-type equation

$$Lu = \begin{cases} u_t - u_{xx} + a_1(t)u(x,0) + a_2(t)u(x,d_1) = 0, & t > 0; \\ u_{tt} - u_{xx} + b_1(t)u(x,0) + b_2(t)u(x,-d_2) = 0, & t < 0, \end{cases} \tag{1}$$

in a rectangular domain $D = \{(x,t) \mid 0 < x < 1, -\alpha < t < \beta\}$, where $a_i(t)$, $b_i(t)$ are given continuous functions, $i = 1, 2$; α, β, d_1, and d_2 are given positive numbers, $0 < d_1 \leq \beta$, $-\alpha < -d_2 < 0$.

Following [1, 2], let us set up the following first boundary problem for Eq. (1).

Problem. In the domain D find a function $u(x,t)$ satisfying the conditions

$$u(x,t) \in C^1(\overline{D}) \cap C^2(D_-) \cap C_x^2(D_+), \tag{2}$$

$$Lu(x,t) = 0, \ (x,t) \in D_- \cup D_+, \tag{3}$$

$$u(0,t) = u(1,t) = 0, \ -\alpha \leq t \leq \beta, \tag{4}$$

$$u(x,-\alpha) = \psi(x), \ 0 \leq x \leq 1, \tag{5}$$

where $\psi(x)$ is a given smooth enough function, $\psi(0) = \psi(1) = 0$,

$$D_- = D \cap \{t < 0\}, \ D_+ = D \cap \{t > 0\}.$$

Note that in [3–9] boundary problems for loaded partial differential equations of separate and mixed types are studied. This is due to the fact that loaded equations find applications in various areas of mathematics and science, e.g., fractal and extreme state physics, mathematical biology and mathematical economics [6–8], in the theory of inverse problems for differential equations [8]. For

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