

ON A CRITERION FOR THE STABILITY OF A SCALAR EQUATION WITH CONSTANT DELAY AND PERIODICAL COEFFICIENT

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Consider the Cauchy problem for the scalar equation

$$\begin{aligned} \dot{x}(t) - a(t)x(t - \omega) &= f(t), \quad t > 0, \\ x(\xi) = 0, \quad \xi < 0; \quad x(0) &= \nu \end{aligned} \tag{1}$$

with a constant delay $\omega > 0$ and an $m\omega$ -periodic coefficient $a(t) : a(t + m\omega) = a(t)$, $m \in \mathbb{N}$. Let, in addition, there be fulfilled the conditions: $a \in L_\infty[0, m\omega]$, $f \in L[0, b] \forall b > 0$ ($L_\infty[0, b]$ is the space of functions $y : [0, b] \rightarrow R$ which are measurable and bounded in global on $[0, b]$, $\|y\|_{L_\infty} = \text{vraisup}_{0 \leq t \leq b} |y(t)|$; $L[0, b]$ is the space of functions $y : [0, b] \rightarrow R$, which are measurable and summable on the segment $[0, b]$, $\|y\|_L = \int_0^b |y(t)| dt$).

In the present article we continue development of ideas and results by Z.I. Rekhbitskii [1], V.V. Malygina [2], [3] on stability of equation (1) in the framework of the method of generating functions (see [4], p. 25). We shall formulate and prove a criterion for the stability of equation (1). We use a boundary value problem, which is satisfied by components of the generating function constructed by the Cauchy function of equation (1) (see [5], p. 115). The idea to utilize the conditions of resolvability of special boundary value problems in studying equations with periodic parameters had arisen long ago (see references in [4]) and was successfully developed in the works of Ye.L. Tonkov, G.I. Yutkin (see, e.g., [6]) (for ordinary differential equations), A.M. Zverkin [7], Yu.F. Dolgiĭ [8], [9] (for equations with delayed argument).

In Z.I. Rekhbitskii's work [1] there was suggested a criterion for stability of equation (1).

Let $a(t)$ be a continuous on $[0, m\omega]$ complex-valued function; in order for all bounded and continuous functions $f(t)$ problem (1) to have bounded solutions $x(t)$ it is necessary and sufficient that all the roots $z_\theta \in \mathbb{C}$ for all $\theta \in [0, \omega]$ of equation $\Delta_m(\theta, z) = 0$ lie outside the unit circle: $|z_\theta| > 1$.

Without writing out an explicit expression for $\Delta_m(\theta, z)$, given in [1], we only note that $\Delta_m(\theta, z)$ is a determinant of m -th order, whose elements are series (entire functions) with respect to z with coefficients depending on the parameter θ . Thus, verification of the condition of the Rekhbitskii's criterion for a concrete equation is a complicated task by itself, not being solved for a sufficiently general case.

It is known from the general theory of functional differential equations (see [5], p.84) that problem (1) has a unique solution

$$x(t) = X(t)\nu + \int_0^t C(t, s)f(s)ds,$$

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