ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2009, Vol. 53, No. 9, pp. 20–28. © Allerton Press, Inc., 2009.
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The Absolutely Representing Families in Certain Classes of Locally Convex Spaces

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Received June 05, 2007

Abstract—A collection $X_\Lambda = \{x_\alpha : \alpha \in \Lambda\}$ of nonzero elements of a complete separable locally convex space H over a scalar field Ψ ($\Psi = \mathbb{R}$ or $\mathbb{C}$), where Λ is a certain set of subscripts, is said to be an absolutely representing family (ARF) in H if $\forall x \in H$ one can find a family in the form $\{c_\alpha x_\alpha : c_\alpha \in \Psi,\ \alpha \in \Lambda\}$ which is absolutely summable to x in H. In this paper we study certain properties of ARF in Fréchet spaces and strong adjoints to reflexive Fréchet spaces. We pay the most attention to obtaining the criteria that allow one to conclude that a given collection X_Λ is an ARF in H.

DOI: 10.3103/S1066369X09090035

Key words and phrases: *absolutely representing families, duality theory, locally convex spaces, Fréchet spaces.*

Dedicated to the memory of Pyotr Lavrent'evich Ul'yanov

1. THE ABSOLUTELY REPRESENTING FAMILIES IN THE FRÉCHET SPACES

1. Let H be a complete separable locally convex space (CSLCS) over a scalar field Ψ, where $\Psi = \mathbb{R}$ or $\Psi = \mathbb{C}$, with a topology determined by a collection of continuous pre-norms $P = \{p\}$. Furthermore, let Λ be a certain collection of subscripts and $x_\alpha \in H$, $x_\alpha \neq 0\ \ \forall \alpha \in \Lambda$. We put $X_\Lambda := \{x_\alpha : \alpha \in \Lambda\}$. According to [1], we call X_Λ an absolutely representing family (ARF) in H if for any element x from H an absolutely summable to x in H family $\{c_\alpha x_\alpha : c_\alpha \in \Psi, \alpha \in \Lambda\}$ exists such that $\sum\limits_{\alpha\in\Lambda} |c_\alpha| p(x_\alpha) < +\infty$ $\forall p \in P$. Probably, the notion of an ARF was first introduced in [1]. In the same paper the author obtained the first criterion for an ARF, i.e., a condition under which a given family X_Λ of elements of H is an ARF in H. This criterion ([1], theorem 1) is based on the concept of a strict X_Λ-topology. See [1] for the definition of this topology, as well as the proof of the mentioned criterion and its certain applications to spaces of analytic functions of one or several variables. However, the criterion established in [1] is not effective enough for determining whether a given family of elements from a CSLCS H is an ARF in the given space.

There exists a more convenient criterion for the Fréchet space H. It is based on the use of the adjoint space H'. This criterion was obtained after the appearance of the paper [1] and was published (with a detailed proof) in [2]. In this section we describe other (equivalent) forms of the criterion proposed in [2] and study their connections with criteria obtained at the same time and even earlier by the author and his successors for a special case with a countable set of subscripts Λ. The corresponding countable sequence $\{x_n\}_{n=1}^{\infty}$ of elements of H was called by the author an absolutely representing system (ARS) in H. These systems were introduced in [3] and studied in papers [4–6] and other ones. The mentioned papers mainly concern the following question: When sequences of usual and generalized exponents are ARS in certain functional spaces? Note that the general notion of an ARS is introduced by the author under the influence of significant works of A. F. Leont'ev summarized in his well-known monograph [7].

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