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# A Nonsymmetrical Problem of the Elasticity Theory for a Ball

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In a spherical system of coordinates $\alpha$, $\beta$, $r$ $(0 \le \alpha \le \pi,\ 0 \le \beta \le 2\pi,\ 0 \le r \le a)$ resolvent equations of the elasticity theory for a ball with respect to displacements are constructed and integrated.

By now several exact solutions to the boundary value problems for an axisymmetrical deformation of a ball are known. Several these problems are described in [1] (see ibid. the list of related references). In the same monograph under certain assumptions one constructs a solution for a nonsymmetrical deformation of a ball. However, it cannot be considered as a general solution, because it contains "extra" constants which are assumed to be zero. This fact has no grounds, therefore we can set these constants to other values. It is interesting, whether the boundary value problem has a unique solution. In our opinion, no essential results are recently obtained in this realm.

In this paper we construct a general solution for a solid or hollow ball; we show a way, enabling one to extend this solution for a segment without poles or for a body bounded by two parallels and meridians.

**1. Construction of resolvent equations.** We use the correlations adduced in [2] with null values of volume forces as the initial data:

$$(\lambda+2\mu)\frac{\partial\theta}{\partial r} - 2\mu\left(\frac{1}{r\sin\alpha}\frac{\partial\omega_\alpha}{\partial\beta} - \frac{1}{r}\frac{\partial\omega_\beta}{\partial\alpha} - \frac{1}{r}\omega_\beta\cot\alpha\right) = 0,$$
$$\frac{(\lambda+2\mu)}{r}\frac{\partial\theta}{\partial\alpha} - 2\mu\left(\frac{\partial\omega_\beta}{\partial r} + \frac{1}{r}\omega_\beta - \frac{1}{r\sin\alpha}\frac{\partial\omega_r}{\partial\beta}\right) = 0, \tag{1}$$
$$\frac{(\lambda+2\mu)}{r\sin\alpha}\frac{\partial\theta}{\partial\beta} - 2\mu\left(\frac{1}{r}\frac{\partial\omega_r}{\partial\alpha} - \frac{1}{r}\omega_\alpha - \frac{\partial\omega_\alpha}{\partial r}\right) = 0.$$

Here $u$, $v$, and $w$ are displacements of ball points along a meridian, a parallel, and in a direction from the center along a radius of the sphere; $E$ and $\nu$ stand for the elasticity module and the Poisson coefficient of the material;

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$

are the Lame coefficients;

$$\theta = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 w) + \frac{1}{r\sin\alpha}\frac{\partial}{\partial\alpha}(u\sin\alpha) + \frac{1}{r\sin\alpha}\frac{\partial v}{\partial\beta} \tag{2}$$

is the volume expansion;

$$\omega_r = \frac{1}{2}\left(\frac{1}{r\sin\alpha}\frac{\partial u}{\partial\beta} - \frac{1}{r}\frac{\partial v}{\partial\alpha} - \frac{1}{r}v\cot\alpha\right), \tag{3}$$
$$\omega_\alpha = \frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{1}{r}v - \frac{1}{r\sin\alpha}\frac{\partial w}{\partial\beta}\right), \quad \omega_\beta = \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial\alpha} - \frac{1}{r}u - \frac{\partial u}{\partial r}\right)$$

are the rotation components.

Let us obtain an equation with respect to $\theta$. To this end, we apply the following operator to the first equation in system (1): $\frac{1}{r^2}\frac{\partial}{\partial r}[r^2(\ )]$; we do the operator $\frac{1}{r\sin\alpha}\frac{\partial}{\partial\alpha}[(\ )\sin\alpha]$ to the second equation, and the