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# GRID APPROXIMATION OF WAVE EQUATION SINGULARLY PERTURBED WITH RESPECT TO SPATIAL VARIABLE

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Solutions of the boundary value problems for partial differential equations with their higher derivatives (or some of them) containing parameter $\varepsilon$ taking arbitrarily small values, possess a bounded smoothness of solutions. For problems of that kind the accuracy of approximate solutions of traditional numerical methods (see, e. g., [1]) depends on the value of the parameter and worsens for small values of $\varepsilon$ (description of the problems arising in the case of elliptic and parabolic equations can be found, e. g., in [2]–[6]). In this connection the problem of development of schemes converging uniformly with respect to the parameter (or, briefly, converging $\varepsilon$-uniformly) arises.

In this article we consider the boundary value problems for singularly perturbed wave equation and also equivalent system of two hyperbolic first order equations in cases of bounded and unbounded domains. In these equations the higher derivative with respect to the spatial variable contains the parameter $\varepsilon$ which takes arbitrary values from the half-segment $(0, 1]$. As $\varepsilon \to 0$, in these problems boundary (hyperbolic) layers appear, which are described by hyperbolic equations. We show that traditional difference schemes do not allow us to obtain approximate solutions whose error does not depend on $\varepsilon$. We show also that in the intrinsic classes of difference schemes there are no adjustment schemes (fitted schemes) which converge $\varepsilon$-uniformly.

For the hyperbolic system with the use of monotone difference approximations of special grids concentrating in boundary layers, schemes are constructed which converge $\varepsilon$-uniformly in the uniform grid norm. For the wave equation the $\varepsilon$-uniform schemes are constructed by "convoluting" schemes for the corresponding hyperbolic system. The convergence rate for such schemes (in the norm of $L_\infty$) is bounded by the value $O(N^{-1} + N_0^{-1})$, where $N+1$ and $N_0+1$ are the numbers of grid nodes by the spatial and time variables, respectively. Note that the norm of $L_p$ and the energetic norm generated by a singularly perturbed wave equation are not adequate for mentioned problems, because the singular part of the solution, which is finite in the uniform norm, tends in these norms to zero as $\varepsilon \to 0$.

## 1. Statement of problems

1. On a set $\overline{G}$, $G = D \times (0, T]$, where $D$ is one of the sets

$$D = (0, d), \tag{1.1}$$
$$D = (0, \infty), \tag{1.2}$$
$$D = (-\infty, 0), \tag{1.3}$$

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