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CONSTRUCTION OF EXPLICIT METHODS OF THE RUNGE–KUTTA TYPE OF INTEGRATION OF SYSTEMS OF THE SPECIAL KIND

I.V. Olemskoi

1. Introduction

In solving the Cauchy problem

$$\begin{gathered} y_i' = f_i(x, y_1, \dots, y_n), \quad i = 1, \dots, n, \\ y_i(X_0) = y_{i0}, \quad i = 1, \dots, n, \\ x \in [X_0, X_1] \subset R, \quad y_i : [X_0, X_1] \longrightarrow R, \quad i = 1, \dots, n, \\ f_i : [X_0, X_1] \times R^n \longrightarrow R, \quad i = 1, \dots, n, \end{gathered} \tag{1.1}$$

the general scheme of the explicit Runge-Kutta method ([1], Chap. 6, § 3, p. 82) of the numerical integration of system (1.1) has the form

$$y_i(x+h) \approx z_i = y_i(x) + \sum_{j=1}^{m_i} b_{ij} k_{ij}(h), \quad i = 1, \dots, n, \tag{1.2}$$

where the functions $k_{ij}(h)$ are calculated by the following scheme:

$$\begin{gathered} k_{ij}(h) = h f_i\Big(x + c_{ij} h, y_1(x) + \sum_{p=1}^{j-1} a_{ij1p} k_{1p}, \dots, y_n(x) + \sum_{p=1}^{j-1} a_{ijnp} k_{np}\Big), \\ c_{i1} = 0, \quad a_{i1s0} = 0, \quad s = 1, \dots, n. \end{gathered} \tag{1.3}$$

Here m_i is the number of stages and q_i is the order with respect to the i-th component of the desired function. In the general case they can be different for each component. So, constructing the computational schemes of the method, their characteristics can be both the vectors, whose components are the number of stages $M = (m_1, \dots, m_n)$ and the order $Q = (q_1, \dots, q_n)$, and the scalars m and q. The first case takes place if at least one component of the corresponding vectors differs from others (then the method is called *multistage* and *multiorder*, respectively). The second case takes place if components of the corresponding vectors equal each other: $m_1 = \ldots = m_n = m$, $q_1 = \ldots = q_n = q$.

By the *order of method* (1.2), (1.3) with the ordering vector $Q = (q_1, \dots, q_n)$ we mean $q = \min\{q_1, \dots, q_n\}$.

Let us introduce one more notion which is necessary for the comparison of the methods of integration of systems of ordinary differential equations.