Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 50, No. 2, pp.55–68, 2006 *UDC 519.632*

THE RICHARDSON METHOD FOR INCREASING THE ACCURACY OF GRID SOLUTIONS OF SINGULARLY PERTURBED ELLIPTIC EQUATIONS OF CONVECTION-DIFFUSION

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On a rectangle we consider the Dirichlet problem for a singularly perturbed elliptic equation with convective terms. For this problem one ε-uniformly convergent monotone difference scheme on piecewise-uniform grids is well known; the order of the rate of its ε-uniform convergence does not exceed one. With the help of solutions of difference schemes on piecewise-uniform embedded grids, using the Richardson method, we construct an approximate solution, converging ε-uniformly with the order which equals two, accurate to a logarithmic multiplier. One can apply the constructions and the substantiation of the schemes, obtaining approximate solutions with a higher precision for three-dimensional problems on a parallelepiped.

1. Introduction

For singularly perturbed boundary value problems special numerical methods are now well developed. As distinct from techniques elaborated for regular boundary value problems (e. g., in [1], [2]), they allow one to obtain grid solutions which converge uniformly with respect to the perturbing parameter ε (or ε-uniformly). In the case of singularly perturbed equations of reaction-diffusion the special difference schemes have the ε-uniform order of convergence rate which is close to two. However, for equations of convection-diffusion the ε-uniform order of convergence rate does not exceed one (see, e. g., [3]–[10] and references therein). Since the efficiency of numerical methods mainly depends on their convergence rate, one needs to construct special schemes for problems of convection-diffusion, whose order of convergence rate exceeds one.

Effective techniques for increasing the accuracy of approximate solutions for regular boundary value problems are the Richardson method and the method of defect correction (or residual correction, see, e. g., [1], [11], [12], and references therein). In singularly perturbed problems of convection-diffusion one applies the method of defect correction to increase the accuracy of approximate solutions with respect to the time variable in nonstationary problems (e. g., [13]) and in spaces variables [14]. In [15], one considers the increase of accuracy of solutions with respect to the space and time for singularly perturbed nonstationary problems of reaction-diffusion with the help of the Richardson method. In [16], one uses the Richardson method to increase the accuracy of solutions of singularly perturbed elliptic equations of convection-diffusion on a strip. Note that the standard defect correction and Richardson methods essentially use grid constructions on uniform grids. However, for sufficiently wide classes of singularly perturbed boundary value problems the use of condensing (at the boundary layer) grids is necessary for the ε-uniform convergence of the schemes (see, e. g., [6], [17]).

The work was supported by the Russian Foundation for Basic Research (project no. 01-01-01022) and, partly, by the Dutch Foundation for Scientific Research NWO (project no. 047.016.008).