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SOLVING THE PROBLEM OF OPTIMAL CONTROL OF INITIAL-BOUNDARY VALUE CONDITIONS OF HYPERBOLIC SYSTEM ON THE BASIS OF EXACT FORMULAS OF INCREMENT

A.V. Arguchintsev

Introduction

The problems of optimal control of systems of first order hyperbolic equations with controllable initial-boundary value conditions given in the form of finite-dimensional connections, possess several principal peculiarities. First, the analogs of the classical pointwise maximum principle are not valid in problems of that form. In [1] for a given class of problems the author proved the variational maximum principle. The optimal boundary or start control supplies the maximum to a special functional in the problem of optimal control of system of ordinary differential equations, which is constructed on characteristics of the initial hyperbolic system. Second, in problems of that type an application of rather effective numerical methods (see [2], [3]), based on the ideas of most capacious approximation of the goal functionals and dynamical systems, is principally impossible. This is implied by the fact that the auxiliary functions of control on every step of respective methods are constructed as functions depending on the state of the process. A generalization of this approach to the hyperbolic systems is possible in the case of distributed controls which enter into right-hand sides of systems (see [3]). In the case of boundary or start controls, the principal difficulty is the non-coincidence of the numbers of independent variables for the functions of control and state.

In this article we investigate a special class of problems of optimal control of linear first order hyperbolic systems, in which the boundary conditions are determined from controllable systems of ordinary differential equations. This nonstandard way to define the boundary value conditions, on one hand, is explained by the desire to overpass the difficulties listed above; on the other hand, by the presence of a series of concrete application problems of this type (see [4]–[6]).

In [7] the validity of the classical maximum principle for this class of problems was shown. In this article, for the sake of simplicity, we consider variants of linear hyperbolic system, linear goal functional, and linear controllable system of ordinary differential equations with the coefficients at phase variables, which depend on the control. The last circumstance (bilinearity of the system of ordinary differential equations) results in the fact that the maximum principle will not be a sufficient optimality condition. The attempt to apply to the problem under consideration more general iteration schemes of the maximum principle (see [8]) results in an infinite iteration process. On every iteration the necessity arises to integrate repeatedly the system of hyperbolic equations.

In this article, on the basis of the application of a nonclassical exact version of the formula of increment, we succeed to reduce the problem to a problem of optimal control of system of ordinary differential equations. In doing so, we have to integrate the initial hyperbolic system only twice:

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