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NONZERO PERIODIC SOLUTIONS OF NONAUTONOMOUS SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

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Let us consider the system of differential equations

$$\dot{x} = A(t,\lambda)x + f(t,x,\lambda)x, \tag{1}$$

in which $x \in E_2$, $A(t,\lambda) = [a_{ij}(t,\lambda)]_1^2$ and $f(t,x,\lambda) = [f_{ij}(t,x,\lambda)]_1^2$ are matrices ω-periodic with respect to t, $\lambda \in E_m$, λ is the parameter, $t \in R =]-\infty,\infty[$, E_s is an s-dimensional vector space.

Let $|x| = \max\limits_i |x_i|$, $D(\delta_0) = \{(x,\lambda) : |x| \le \delta_0,\ |\lambda - \lambda_0| \le \delta_0\}$, $W(\delta_0) = \{\alpha : \alpha \in E_2,\ |\alpha| \le \delta_0\}$, $\Lambda(\delta_0) = \{\lambda : \lambda \in E_m,\ |\lambda - \lambda_0| \le \delta_0\}$, $\delta_0 > 0$ being a certain number, $\lambda_0 \in E_m$.

We determine the conditions of existence of a nonzero periodic solutions of system (1). Such a problem was studied in [1] by the technique of small parameter. In [2] the problem on the existence of a periodic solution of a system of differential equations under the condition of existence of Green's function in a certain generalized sense was reduced to an analogous problem for a system of integral equations; the principle of contracting mappings was suggested for investigation of this system. In [3], for a system of differential equations whose right-hand side is a holomorphic vector function with respect to the variables x, λ, in the conditions of fulfillment of a bifurcation system of equalities, a theorem on the existence of a periodic solution was proved. The problem of determination of the conditions for existence of a periodic solution was reduced in [4] to the problem on solvability of transcendent equations of bifurcation.

In this article we prove a theorem on the conditions of existence of nonzero periodic solution of system (1), which are defined by rather general dependence of elements of the matrix of system of linear approximation on the parameter; in this situation, it is assumed that the nonlinear terms should be simply continuous, while system (1) possesses the property of the uniqueness of solution.

Everywhere in what follows we assume that on a set $R \times D(\delta_0)$ the matrices $A(t,\lambda)$ and $f(t,x,\lambda)$ are continuous, $f(t,0,\lambda) \equiv 0$.

By means of the change of variables

$$x_1 = \Big(\exp \int_0^t a_{11}(\tau,\lambda)d\tau\Big)y_1, \quad x_2 = \Big(\exp \int_0^t a_{22}(\tau,\lambda)d\tau\Big)y_2 \tag{2}$$

we reduce system (1) to the system

$$\dot{y} = B(t,\lambda)y + \Psi(t,y,\lambda)y, \tag{3}$$