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A Majorant Criterion for the Total Preservation of Global Solvability of Controlled Functional Operator Equation

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Abstract—We prove the unique solvability of a nonlinear controlled functional operator equation in a Banach ideal space. We also establish sufficient conditions for the global solvability of all controls from a pointwise bounded set, provided that some majorant equation for the given family of these controls is globally solvable. We give examples of controlled boundary value problems reducible to the considered equation.

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INTRODUCTION

Let use the following denotations: $n, m, \ell, s \in \mathbf{N}$ are given numbers, $\Pi \subset \mathbf{R}^n$ is a measurable (hereinafter in the Lebesgue sense) bounded set, $\mathcal{X}$, $\mathcal{Z}$, and $\mathcal{U}$ are *Banach ideal spaces*[1)] (BIS) of measurable on Π functions, $\mathcal{D} \subset \mathcal{U}^s$ is a convex set, $A : \mathcal{Z}^m \to \mathcal{X}^\ell$ is a given *linear bounded operator* (LBO). Further for a vector function $x \in \mathcal{X}^\ell$ that is the image of a vector function $z \in \mathcal{Z}^m$ under the mapping realized by the operator A, depending on the situation, we use the following equivalent denotations: $x = A[z]$; $x(t) = A[z](t)$, $t \in \Pi$; $x(\cdot) = A[z](\cdot)$; and $x = A[z(\cdot)]$. Consider the following controlled functional operator equation:

$$x(t) = \theta(t) + A\big[f(\cdot, x(\cdot), u(\cdot))\big](t), \quad t \in \Pi, \quad x \in \mathcal{X}^\ell, \tag{0.1}$$

where $u \in \mathcal{D}$ is the control, $\theta \in \mathcal{X}^\ell$, and $f : \Pi \times \mathbf{R}^\ell \times \mathbf{R}^s \to \mathbf{R}^m$ is a given function such that $\mathbf{F}_1$) for all $y \in \mathcal{X}^\ell$ and $u \in \mathcal{D}$ the superposition $f\big(\cdot, y(\cdot), u(\cdot)\big)$ belongs to $\mathcal{Z}^m$.

A rather wide class of controlled *initial-boundary value problems* (IBVP) can be reduced to Eq. (0.1) by *conversion of the main part* of the differential equation. Let us clarify this with the help of the following example, which is classical in the theory of optimization of distributed systems, namely, the controlled Goursat–Darboux problem

$$\begin{cases} x''_{t_1 t_2}(t) = f\big(t, x(t), u(t)\big), & t = (t_1, t_2) \in \Pi = [0, T_1] \times [0, T_2]; \\ x(t_1, 0) = \omega_1(t_1), & t_1 \in [0, T_1]; \\ x(0, t_2) = \omega_2(t_2), & t_2 \in [0, T_2]; \\ \omega_1(0) = \omega_2(0). \end{cases} \tag{0.2}$$

We assume that functions ω_1 and ω_2 are absolutely continuous and the function f satisfies the condition $\mathbf{F}_1$) with $\ell = 1$, $m = 1$, $\mathcal{X} = L_q(\Pi)$, $\mathcal{Z} = L_p(\Pi)$, $1 \le p < q \le +\infty$, and $U = L_r(\Pi)$, $r \in [1, \infty]$. We understand a solution to problem (0.2) in the almost everywhere (a. e.) sense and seek for it among

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[1)] Recall that a Banach space E of measurable functions is said to be a Banach ideal space, if $\{y \in E,\ x \text{ is a measurable function}, |x| \le |y|\} \Longrightarrow \{x \in E, \|x\|_E \le \|y\|_E\}$.