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Nonlocal Problem with Integral Conditions for a System of Hyperbolic Equations in Characteristic Rectangle

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Abstract—We consider a nonlocal problem with integral conditions for a system of hyperbolic equations in rectangular domain. We investigate the questions of existence of unique classical solution to the problem under consideration and approaches of its construction. Sufficient conditions of unique solvability to the investigated problem are established in the terms of initial data. The nonlocal problem with integral conditions is reduced to an equivalent problem consisting of the Goursat problem for the system of hyperbolic equations with functional parameters and functional relations. We propose algorithms for finding a solution to the equivalent problem with functional parameters on the characteristics and prove their convergence. We also obtain the conditions of unique solvability to the auxiliary boundary-value problem with an integral condition for the system of ordinary differential equations. As an example, we consider the nonlocal boundary-value problem with integral conditions for a two-dimensional system of hyperbolic equations.

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1. INTRODUCTION

We consider a nonlocal boundary-value problem with integral conditions for a system of hyperbolic equations with mixed derivatives in a rectangular domain $\overline{\Omega} = [0,T] \times [0,\omega]$:

$$\frac{\partial^2 u}{\partial t \partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + f(t,x), \tag{1.1}$$

$$\int_0^a K(t,x)u(t,x)dx = \psi(t), \quad t \in [0,T], \tag{1.2}$$

$$\int_0^b K(t,x)u(t,x)dt = \varphi(x), \quad x \in [0,\omega], \tag{1.3}$$

where $u(t,x) = \mathrm{col}(u_1(t,x), u_2(t,x), \ldots, u_n(t,x))$ is an unknown function, $(n \times n)$-matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, n-vector-function $f(t,x)$ are continuous on $\overline{\Omega}$, $(n \times n)$-matrix $K(t,x)$ is continuously differentiable on $\overline{\Omega}$, and n-vector-functions $\varphi(x)$, $\psi(t)$ are continuously differentiable on $[0,\omega]$, $[0,T]$, respectively, $0 < a \le \omega$, $0 < b \le T$. We assume that functions $\varphi(x)$ and $\psi(t)$ satisfy the concordance condition $\int\limits_0^a \varphi(x)dx = \int\limits_0^b \psi(t)dt$.

Nonlocal problems with integral conditions for equations of hyperbolic type, questions of their existence and the uniqueness of their solutions were considered in papers [1–22]. One established conditions of classical generalized resolvability of problems with integral conditions for hyperbolic

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