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METHOD OF UNITARY TRANSFORMS IN THE THEORY OF STABILITY

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In the study of stability of solution of nonautonomous linear, quasilinear, and nonlinear systems of ordinary differential equations, in contrast to linear homogeneous systems with a constant matrix (when the behavior of their solution is completely determined by the spectrum of the matrix of the system), a large number of problems arise, which are studied only partly, for example, in [1]–[11].

A series of theorems are known (see [2], [6], [7]) which make it possible to investigate the stability of the trivial solution of nonautonomous systems for "small" perturbations of the constant matrix of system or in the presence of "small" nonlinear inhomogeneities in systems with constant matrix (for example, the Lyapunov theorem on asymptotic stability by first approximation).

A certain class of "reducible" linear nonautonomous systems can be singled out, for which a nondegenerate transform exists (and in some cases can be constructed) (see [1]–[7]), which reduces the initial system to a system with constant matrix (for example, the Floquet–Lyapunov theorem on reducibility of periodic linear systems is known). Let us note the result by Wazewski (see [2], [7]) which is related to a linear nonautonomous system whose matrix commutes with the integral of this matrix. In this situation, the behavior of such a system (as in the case of the system with constant matrix) is completely determined by the spectrum of the matrix of system. However, for an arbitrary linear nonautonomous system this fact takes no place.

Example 1. The trivial solution of the system

$$\dot{x} = A(t)x, \quad A(t) = \begin{pmatrix} -1 - 2\cos 4t & -2 + 2\sin 4t \\ 2 + 2\sin 4t & -1 + 2\cos 4t \end{pmatrix}$$

is unstable; that was proved in [9] (p. 123) by means of the Lyapunov indicators, though eigenvalues of the matrix $A(t)$, which are equal to $\lambda_{1,2} = -1$, lie in the left halfplane.

Example 2. For a linear system with the periodic matrix

$$\dot{x} = A(t)x, \quad A(t) = \begin{pmatrix} \cos\omega t & \sin\omega t \\ \sin\omega t & -\cos\omega t \end{pmatrix},$$

which was suggested to the author by Prof. A.F. Filippov, it became possible to prove (in the presence of constant eigenvalues $\lambda_{1,2} = \pm 1$ of the matrix $A(t)$) that the behavior of the solution of system as $t \to +\infty$ depends essentially on the frequency ω. A nondegenerate unitary change was constructed

$$x = U(t)y, \quad U(t) = \begin{pmatrix} -\sin\frac{\omega t}{2} & \cos\frac{\omega t}{2} \\ \cos\frac{\omega t}{2} & \sin\frac{\omega t}{2} \end{pmatrix},$$

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