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A CONSTRUCTIVE INVESTIGATION OF DIFFERENTIAL EQUATIONS WITH ARBITRARY DEVIATION OF ARGUMENT

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The present article continues the series of works which develop constructive methods in the theory of functional differential equations (see [1]–[5]). The objective of the present investigation is a linear differential equation with arbitrary deviation of argument.

It is appropriate to mention a number of works of foreign authors concerning this subject. The papers [6], [7] are devoted to construction of constructive methods for investigation of ordinary differential and integral equations. In [8], there were considered interval methods for analysis of operator equations. In [9], constructive methods were proposed for investigation of the resolvability of nonlinear boundary value problems for ordinary differential equations.

In the introductory Section we give brief survey of necessary information concerning the theory of functional differential equations. Then we describe a ground of a constructive computer-oriented method for investigation of existence of solution of a differential equation with arbitrary deviation of argument. In conclusion we give an illustrative example.

1. Notation

Everywhere below, R stands for the space of real values with the norm $|\cdot|$; L is the space of summable functions $z : [0,T] \to R$ with the norm $\|z\|_L = \int_0^T |z(s)|ds$; L_∞ means the space of essentially bounded functions $z : [0,T] \to R$ with the norm $\|z\|_{L_\infty} = \operatorname{vrai\,sup}_{t\in[0,T]} |z(t)|$; D is the space of absolutely continuous functions $x : [0,T] \to R$ with the norm $\|x\|_D = |x(0)| + \|\dot{x}\|_L$, and $S_h : D \to L$ is an operator of the inner superposition:

$$(S_h)(x) = \begin{cases} x[h(t)], & \text{if } h(t) \in [0,T]; \\ 0, & \text{if } h(t) \notin [0,T], \end{cases}$$

If the function $\phi(\xi)$ is defined for $\xi \notin [0,T]$, then

$$(\phi^h)(t) = \begin{cases} 0, & \text{if } h(t) \in [0,T]; \\ \phi[h(t)], & \text{if } h(t) \notin [0,T]. \end{cases}$$

2. Introduction

Let us give the necessary information concerning the theory of linear functional differential equations [1].

We consider the following equation

$$(\mathcal{L}x)(t) = f(t), \quad t \in [0,T], \tag{1}$$