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COMPARING TWO REGULARIZING ALGORITHMS FOR SOLVING AN INVERSE COEFFICIENT PROBLEM

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We consider a conditionally well-posed problem of determination of the lowest coefficient of a parabolic equation, when this coefficient does not depend on time. Earlier, for solving this problem, the quasi-inversion method was used. In this article we formulate the variational statement of the problem. To solve it we suggest to use the regularization method. We prove a theorem stating that in a particular case the regularization algorithm can be reduced to the investigation of the corresponding problem of quasi-inversion. As a consequence of this result, within the continuation of solving the parabolic equation, when the overdetermined boundary value conditions are given on the whole boundary, the problem of quasi-inversion coincides with a particular case of the regularized variational problem.

1. *Statement of the problem of determination of the lowest coefficient in a parabolic equation.* Assume that the coefficient $q(x)$ at the lowest term of the parabolic equation

$$u_t - u_{xx} = q(x)u \tag{1.1}$$

is unknown. Following [1]–[4], we formulate the problem of determination of this coefficient as the inverse coefficient problem (see [5], p. 16): determine a vector function $\{q(x), u(x,t)\}$ from the conditions

$$\begin{aligned} &u_t - u_{xx} = q(x)u, \quad 0 < x < 1, \quad t > 0;\\ &u(0,t) = f_0(t), \qquad u_x(0,t) = f_1(t),\\ &u(1,t) = g_0(t), \qquad u_x(1,t) = g_1(t), \quad t > 0;\\ &u(x,0) = u_0(x), \qquad 0 \le x \le 1. \end{aligned}$$

Let us transform equation (1.1) to a form not containing the coefficient $q(x)$. To this end we solve equation (1.1) with respect to $q(x)$ and then, taking into account that $q_t = 0$, differentiate the result with respect to the variable t:

$$\begin{aligned} q &= (u_t - u_{xx})/u,\\ u_{tt}u - u_{xxt}u - u_t^2 + u_{xx}u_t &= 0. \end{aligned} \tag{1.2}$$

In equation (1.2) we pass from the function $u(x,t)$ to the new unknown function $v = u_t/u$. Taking into account the relations

$$\frac{u_{tt}}{u} = v^2 + v_t, \quad \frac{u_{xxt}}{u} = \frac{u_{xx}}{u}v + 2\frac{u_x}{u}v_x + v_{xx}, \quad \frac{u_x}{u} = \int_0^t v_x dt + \frac{u_{0x}}{u_0},$$

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