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# Tracking of Solutions to Parabolic Equations with Memory in a General Class of Controls

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**Abstract**—We consider a control problem for a parabolic equation with memory. It consists in constructing an algorithm for finding a feedback control which enables one to track a solution of the given equation with an unknown right-hand side. For this problem we propose two noise-resistant solution algorithms based on the method of extremal shift. The first algorithm is applicable in the case of continuous measurements of phase states, whereas the second one presumes discrete measurements.

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## 1. INTRODUCTION. THE PROBLEM

Let $(V, |\cdot|_V)$ be a separable reflexive Banach space densely and continuously embedded in a Hilbert space $(H, |\cdot|_H)$. Identifying $H$ with its conjugate space $H^*$, we get embeddings $V \subset H \subset V^*$. Let the symbol $(\cdot,\cdot)$ denote the duality between $V$ and $V^*$, as well as the scalar product in $H$.

Consider the tracking problem for a solution to the equation

$$\dot{x}(t) + Ax(t) + \int_0^t f(t-s)Ax(s)\,ds = Bu(t), \quad t \in T = [0, \vartheta], \tag{1.1}$$

where $x \in H$, $H$ is the phase space; $x(0) = x_0 \in V$ is the initial state; the memory function $f(\cdot) \in W_2^1(T, \mathbb{R}) = \{f(\cdot) \in L_2(T;\mathbb{R}), \dot{f}(\cdot) \in L_2(T;\mathbb{R})\}$; $B$ is a linear continuous operator acting from the Hilbert space $U$ (the space of controls) with the norm $|\cdot|_U$ and the scalar product $(\cdot,\cdot)_U$ to the space $H$. We understand the derivative $\dot{x}$ in the sense of distributions ([1], P. 14). The operator $A : V \to V^*$ satisfies the following conditions:

(A1) $A$ is hemicontinuous, i.e., the map $t \to (A(u + tv), w)$ is continuous on $T$ for all $u, v, w \in V$;

(A2) $A$ is strongly monotone, i.e., there exists $\omega > 0$ such that the inequality $(Au - Av, u - v) \geq \omega|u - v|_V^2$ is valid for all $u, v \in V$;

(A3) there exists $b_1 > 0$ such that $\|Av\|_{V^*} \leq b_1|v|_V$ for all $v \in V$.

**Remark.** Equations in the form (1.1) were studied in papers [2, 3], as well as in [4, 5], where one considered such issues as the existence, uniqueness, and regularity of solutions. The importance of this research is connected with the fact that the well-known in the theory of materials with memory equation

$$\dot{x}(t,\eta) + \Delta x(t,\eta) + \int_0^t f(t-s)\Delta x(s,\eta)\,ds = u(t,\eta), \quad x|_\Gamma = 0,$$

where $\eta \in \Omega \subset \mathbb{R}^n$, $t \in T$, is a particular case of Eq. (1.1).

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