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Analytic Solutions to Heat Transfer Problems on a Basis of Determination of a Front of Heat Disturbance

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Abstract—With the use of additional boundary conditions in integral method of heat balance, we obtain analytic solution to nonstationary problem of heat conductivity for infinite plate. Relying on determination of a front of heat disturbance, we perform a division of heat conductivity process into two stages in time. The first stage comes to the end after the front of disturbance arrives the center of the plate. At the second stage the heat exchange occurs at the whole thickness of the plate, and we introduce an additional sought-for function which characterizes the temperature change in its center. Practically the assigned exactness of solutions at both stages is provided by introduction on boundaries of a domain and on the front of heat perturbation the additional boundary conditions. Their fulfillment is equivalent to the sought-for solution in differential equation therein. We show that with the increasing of number of approximations the accuracy of fulfillment of the equation increases. Note that the usage of an integral of heat balance allows the application of the given method for solving differential equations that do not admit a separation of variables (nonlinear, with variable physical properties etc.).

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It is known that the exact analytical solutions to the heat conductivity problems, constructed through classical analytical techniques and presented in the form of infinite series, do not have good convergence at small values of the time coordinate. Computations show that the convergence of the heat conductivity problem exact analytical solution for an infinite plate and the first kind boundary conditions in the range of Fourier numbers $10^{-12} \le \mathrm{Fo} \le 10^{-7}$ happen only if we take from 2000 ($\mathrm{Fo} = 10^{-7}$) to five hundred thousand ($\mathrm{Fo} = 10^{-12}$) series summands ([1], P. 102).

Variational methods (Ritz, Trefftsa et al.) as well as suspended residuals methods (Bubnov–Galerkin, L. V. Kantorovich, etc.) are practically inapplicable for solutions with small time values since in order to determine the eigenvalues of boundary-value problems we need to solve algebraic equations of higher degrees, and the implementation of the initial conditions is connected with the solving of large systems of linear algebraic equations with bad conditioned coefficient matrices.

The analytical heat conductivity theory contains methods which introduce the thermal perturbation front concept (heat balance integral method) [1–13]. Under application of these methods we formally divide the body heating (cooling) process into two stages. The first of these is characterized by the gradual advancement of the front perturbations from the surface to the center, and the second implies the body temperature change throughout the whole solid volume. This thermal conductivity model belongs to a number of methods: the integral method of heat balance [2–5], the method of averaging functional corrections [7], the methods of M. E. Swetz [6], M. Biot [4], A. I. Veinik [5] and others. Their advantage is the possibility to obtain simple in shape approximate analytical solutions, their significant drawback is low accuracy. The reason of this accuracy is that the resulting solution precisely meeting the initial

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