Multidimensional Gravity. Advantages and disadvantages

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Recent future:

- Parameters of the Higgs field are determined.
- Neutrino physics (mixing angles, masses) is known.
- Enigma of the Dark Matter is resolved
- The Universe evolution is studied (more or less)

Is physics finished???

Something remains.

Future theory should:

1. Explain the origin of fields, symmetries, number of generations

2. Explain numerical values of the physical parameters

3. Solve the fine tuning problem and the hierarchy problem
…………………………..

Most economical basis – multidimensional gravity

Minimal number of initial ideas is attracted:

1. **Space time foam at the Planck scale.**

2. **Gravity with higher derivatives**
$L \sim R + aR^2 \ldots$

Inevitable consequences of the gravity and the quantum theory

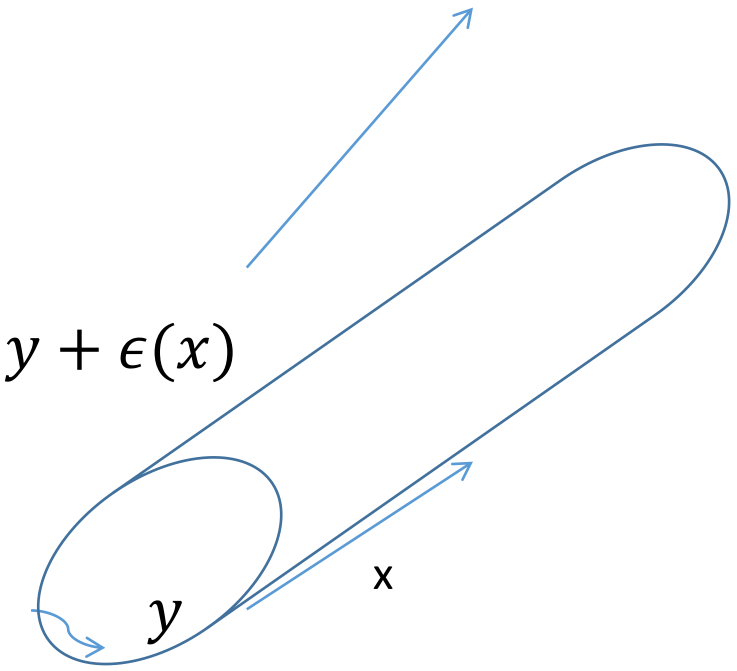
3. **Extra dimensions** – *new postulate*

CONTENT

Electromagnetic field as off–diagonal components of 5-dim space. Kaluza-Klein model

$$A'_\mu = A_\mu - \partial_\mu \epsilon$$

$$y' = y + \epsilon(x)$$



A,B=1,2,3,4,5

$$g_{AB} = \left[\begin{array}{c|c} g_{\mu\nu} & \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \\ \hline A_1\ A_2\ A_3\ A_4 & \varphi(x) \end{array}\right]$$

Gauge symmetries is the result of appropriate symmetry of an extra space

D-dim Riemann space

$$ds^2 = g_{AB} dx^A dx^B,\ A,B = 1,2,\dots,D,\ D > 4.$$

$$g_{ab}(x,\{\lambda\})$$

$$a,b = 4,..,D$$

$$g_{AB} = \left[\begin{array}{c|c} g_{\mu\nu} & g_{\mu a} \\ \hline g_{a\mu} & g_{ab} \end{array}\right]$$

Vector fields $A_\mu \dots$

Scalar fields $\varphi_i \dots$

Transition to observed 4-dim world

$$S[g_{AB}(x,y_{extra}),\{\lambda\}] \rightarrow S_{obs}\left[g_{\mu\nu}(x), A_\mu(x), \varphi_i(x), \{\lambda\}\right]$$

Инвариантность g_{ab} при координатных преобразованиях, индуцированных группой G, приводит к инвариантности Лагранжиана относительно этой группы и появлению калибровочных полей.

Important: an effective Lagrangian depends on an extra space metric

How to manage F(R) D-dim gravity?

First way

$$S = \frac{1}{2\kappa_D^2} \int_M d^D x \sqrt{|\bar{g}|} f(\bar{R})$$

$$f' \bar{R}_{ab} - \frac{1}{2} f \, \bar{g}_{ab} - \bar{\nabla}_a \bar{\nabla}_b f' + \bar{g}_{ab} \bar{\Box} f' = 0$$

Reduction to linear gravity

$$g_{ab} = \Omega^2 \bar{g}_{ab} = \left[f'(\bar{R}) \right]^{2/(D-2)} \bar{g}_{ab}$$

$$S = \frac{1}{2\kappa_D^2} \int_M d^D x \sqrt{|g|} \left[R[g] - g^{ab} \phi_{,a} \phi_{,b} - 2U(\phi) \right] \; ,$$

where

$$f'(\bar{R}) = \frac{df}{d\bar{R}} := e^{A\phi} > 0 \; , \quad A := \sqrt{\frac{D-2}{D-1}} \; ,$$

and where the self-interaction potential $U(\phi)$ of the scalar field ϕ is given by

$$\begin{aligned} U(\phi) &= \frac{1}{2} (f')^{-D/(D-2)} \left[\bar{R} f' - f \right] \; , \\ &= \frac{1}{2} e^{-B\phi} \left[\bar{R}(\phi) e^{A\phi} - f\left(\bar{R}(\phi) \right) \right] \; , \quad B := \frac{D}{\sqrt{(D-2)(D-1)}} \; . \end{aligned}$$

Second way

$$S = \frac{1}{2} m_D^{D-2} \int \sqrt{{}^D g}\, d^D x\, [F(R) + L_m] \qquad ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + e^{2\beta(x)} h_{ab} dx^a dx^b$$

$$\begin{aligned} R &= R_4 + \phi + f_{\rm der}, \qquad |\phi| \gg |R_4|,\ |f_{\rm der}| \\ \phi &= kd(d-1) m_D^2\, e^{-2\beta(x)} \\ f_{\rm der} &= 2d g^{\mu\nu} \nabla_\mu \nabla_\nu \beta + d(d+1)(\partial\beta)^2, \qquad F(R) = F(\phi + R_4 + f_1) \simeq F(\phi) + F'(\phi)\cdot(R_4 + f_1) + \ldots, \end{aligned}$$

$$\begin{aligned} S &= \frac{\mathcal{V}[d_1]}{2} m_D^2 \int \sqrt{\tilde g}\, (\mathrm{sign}\, F') L, \\ L &= \tilde R_4 + \frac{1}{2} K_{\rm Ein}(\phi)(\partial\phi)^2 - V_{\rm Ein}(\phi) + \tilde L_m, \end{aligned}$$

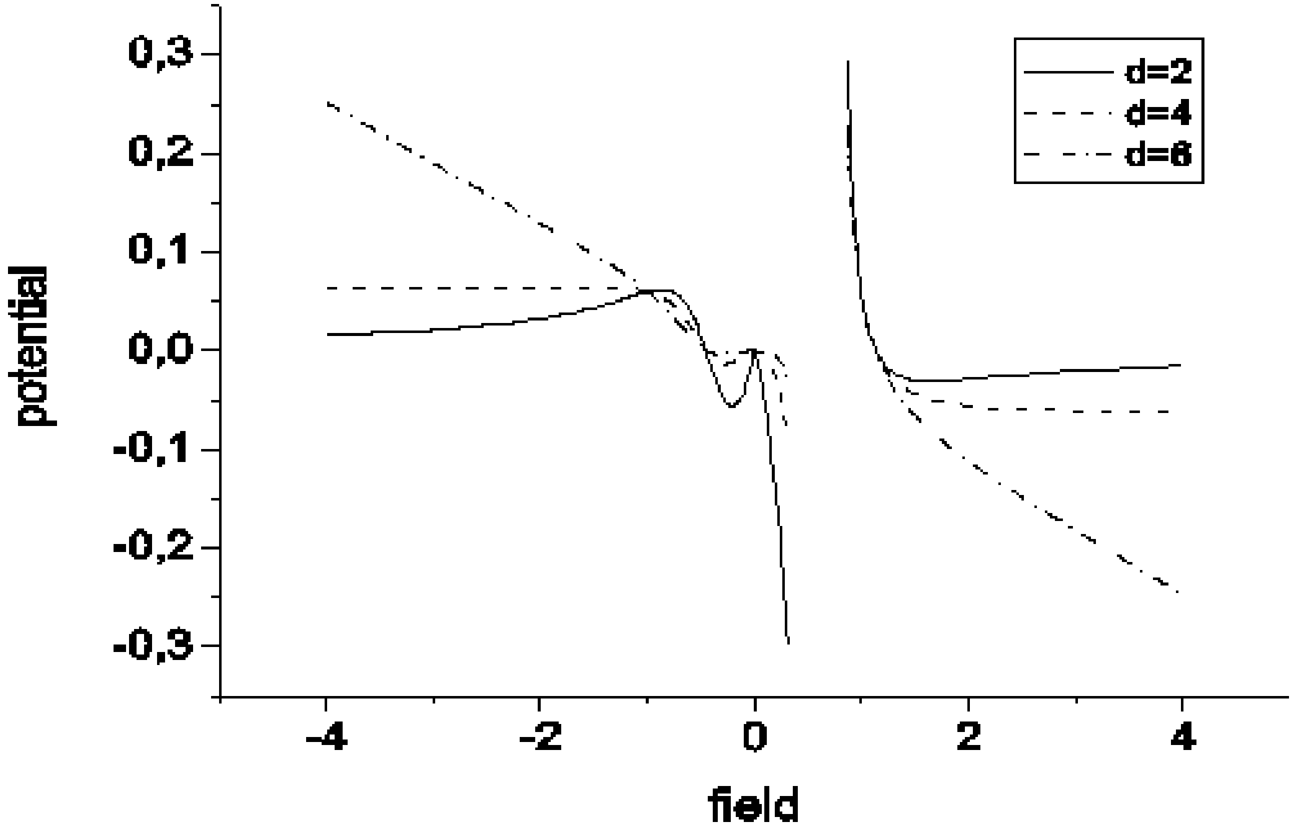
$$\tilde L_m = (\mathrm{sign}\, F') \frac{e^{-d_1\beta}}{F'(\phi)^2} L_m; \qquad (15)$$

$$K_{\rm Ein}(\phi) = \frac{1}{2\phi^2} \left[6\phi^2 \left(\frac{F''}{F'} \right)^2 - 2d_1 \phi \frac{F''}{F'} + \frac{1}{2} d_1 (d_1 + 2) \right], \qquad (16)$$

$$V_{\rm Ein}(\phi) = -(\mathrm{sign}\, F') \left[\frac{|\phi| m_D^{-2}}{d_1(d_1 - 1)} \right]^{d_1/2} \frac{F(\phi)}{F'(\phi)^2} \qquad (17)$$

Stability of extra space

$$F(R) = R + cR^2 + w_1R^3 + w_2R^4 - 2\Lambda$$



$$l \sim \phi(x)^{-1}$$

Tunneling between two minima?

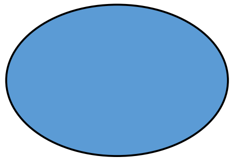
B. Origination of gauge symmetries. General case

Extra space MUST be symmetrical (gauge symmetries do exist)

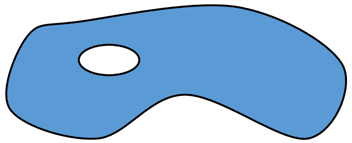
but

Why extra space IS symmetrical indeed?

case A: one state



case B: many states



Why P(A)>P(B)?

Note: The set of all geometries is infinite, hence formation probability of specific, e.g. symmetric geometry is small

It means that some mechanism of symmetrization must exist !

Preliminary result

$$S = \int d^{D+1}z\sqrt{G}f(R), \qquad ds^2 = G_{AB}dX^A dX^B = dt^2 - g_{mn}(t,x)dx^m dx^n - \gamma_{ab}(t,x,y)dy^a dy^b$$

After conformal transformation of metric G

$$S = \int d^{D+1}z\sqrt{\tilde{G}}\left[\tilde{R}(\tilde{G}) + \tilde{G}^{ab}\partial_a\phi\partial_b\phi - 2U(\phi)\right],$$

where

$$\phi = \frac{1}{A}ln f'(R); \quad A = \sqrt{\frac{D-1}{D}}$$

$$U(\phi) = \frac{1}{2}e^{-B\phi}\left[R(\phi)e^{A\phi} - F(R(\phi))\right], \quad B = \frac{D+1}{\sqrt{(D-1)D}}$$

$$\ddot{\phi} + 3H\dot{\phi} + \Box_{d+1}\phi + U'(\phi) = 0.$$

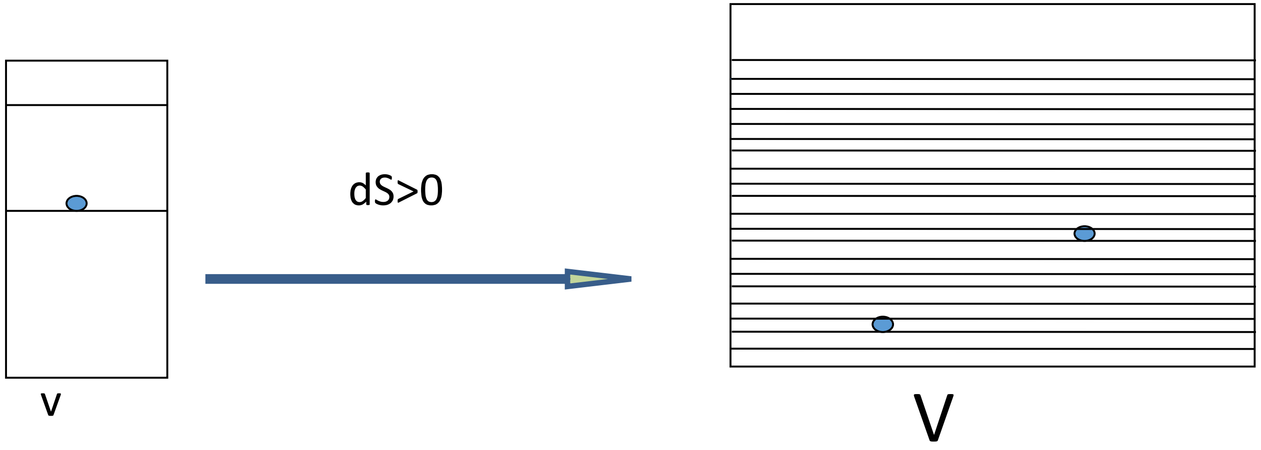
Friction due to the contact with 4-dim space is the reason of the scalar field (the Ricci scalar of the extra space) tends to a potential minimum.

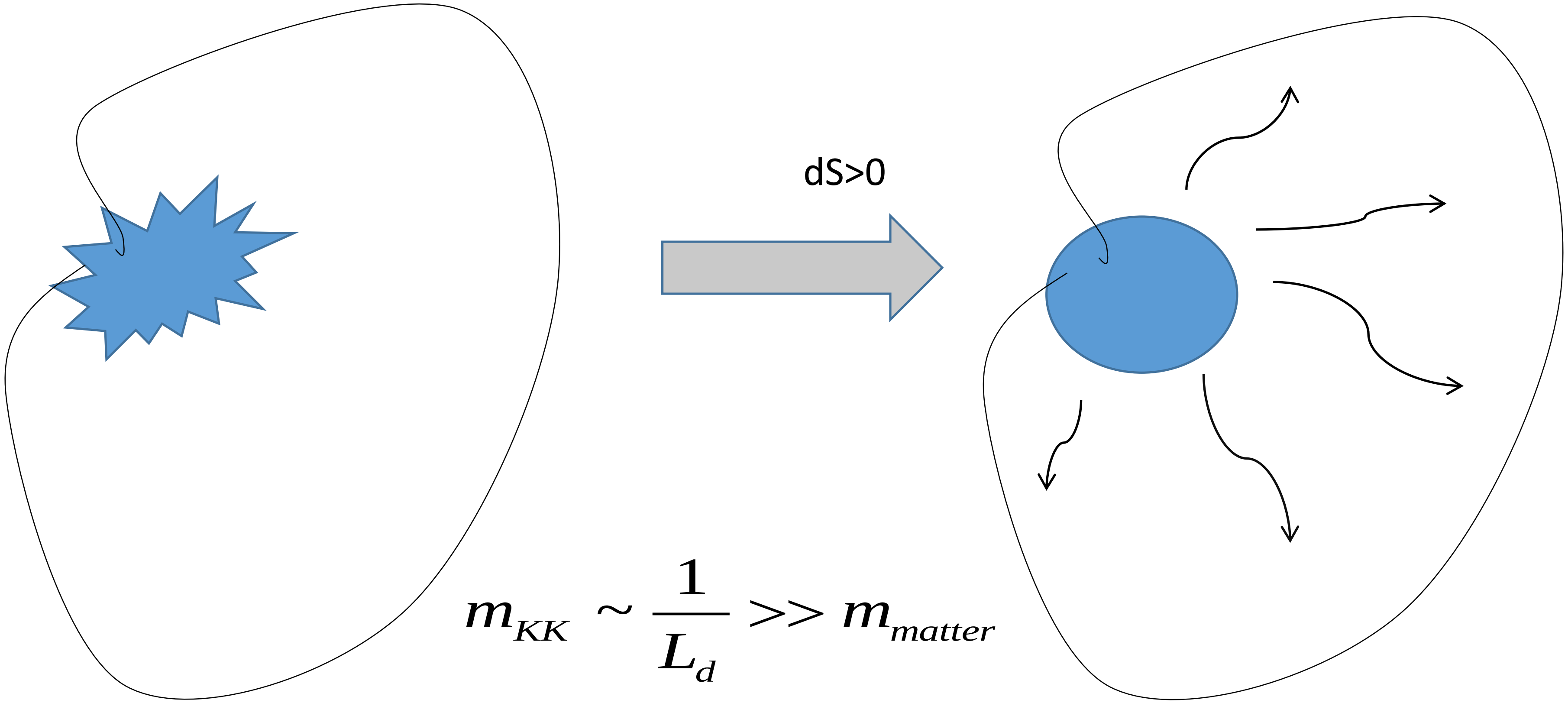
$\phi \rightarrow const$ → $R \rightarrow const$

Final state: Maximally symmetric and stationary extra space

How could one produce symmetrical extra space?

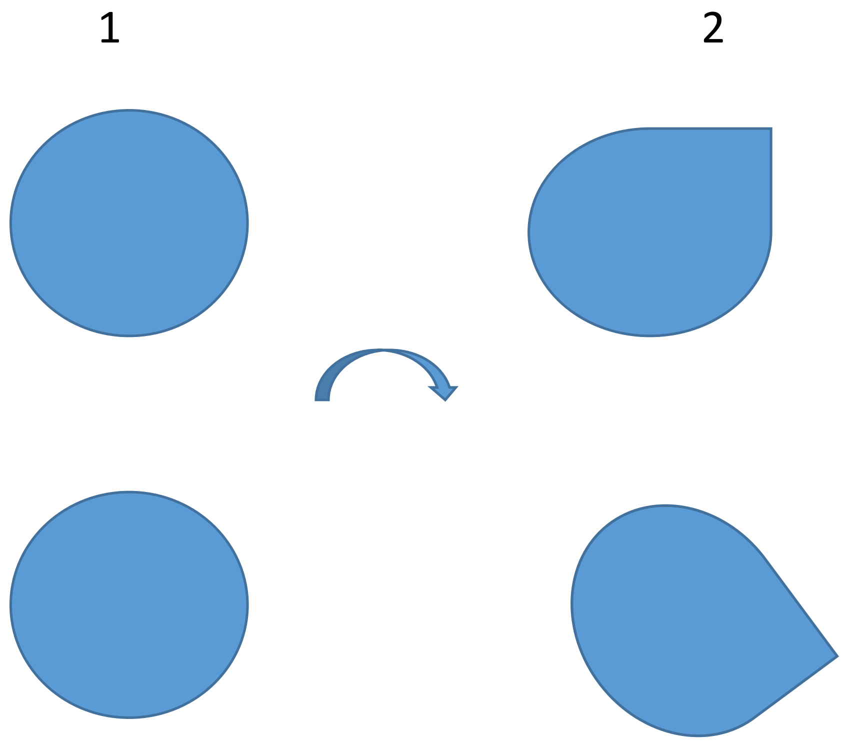
Main idea





$$m_{KK} \sim \frac{1}{L_d} >> m_{matter}$$

Less entropy --- more symmetry.



$$\Omega_1 < \Omega_2$$

Extra space is nucleated being asymmetrical.
Its entropy tends to minimum due to KK particle decay.
Hence the extra space tends to maximally symmetrical geometry.

Now we understand that gauge symmetries are formed due to entropy growth

Immediate conclusion

- Gauge symmetries are absent at the beginning of inflation

They are formed during inflation or after it.

Charges are not conserved in the early Universe!

When does it happens – before inflation? During inflation or after it?

Problem of the baryon asymmetry

Baryon asymmetry of the Universe is challenging problem for a long time.
How to explain nonconservation of baryon charge in early Universe

and

at the same time the proton stability nowadays??

In extra space approach, this is almost trivial fact:

From the beginning – no symmetries → **no charge conservation**

During the Universe evolution, the entropy is growing that lead to extra space symmetrization

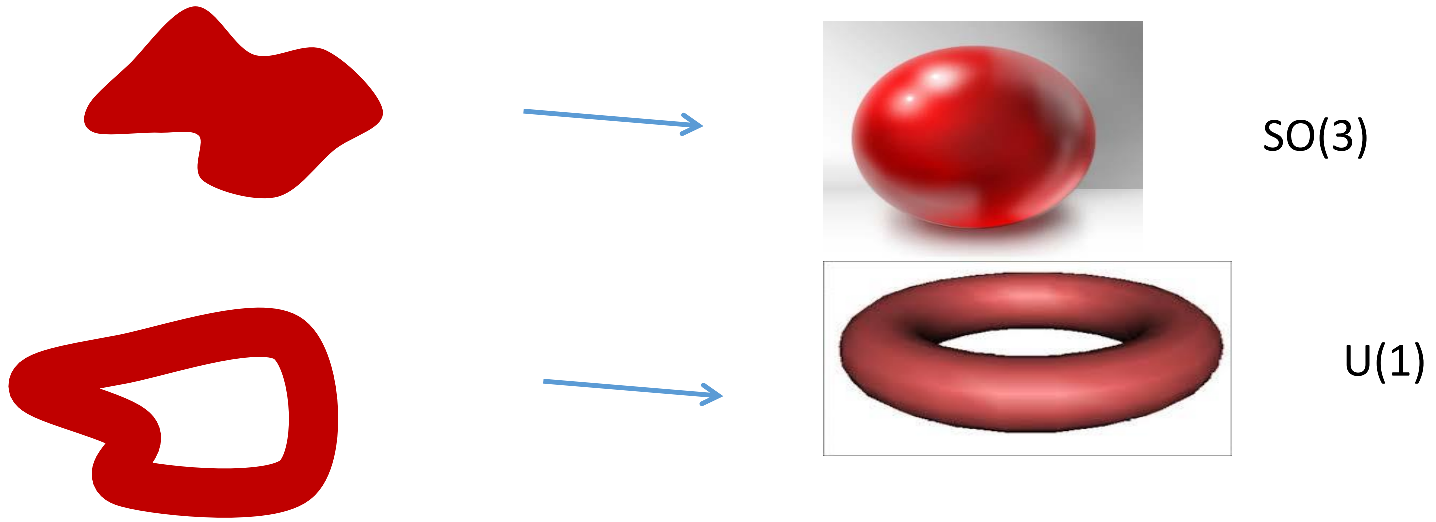
Baryon asymmetry of the Universe is **inevitable** in such approach

The charge asymmetry is also inevitable! Slightly charged Universe???

What a mechanism that prevent an extra space to be maximally symmetrical?

Geometry of ES and excitation spectrum depends on:

1. Topology of ES
2. Boundary conditions



What is the extra space metric that lead to the Standard Model?

Role of boundary conditions.

(1) $$S = \frac{m_D^{D-2}}{2}\int d^4x d^n y \sqrt{|G(y)g(x)|} f(R), \quad f(R) = \sum_k a_k R^k$$

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + G_{ab}(x,y)dy^a dy^b.$$

Classical equations derived from (1)

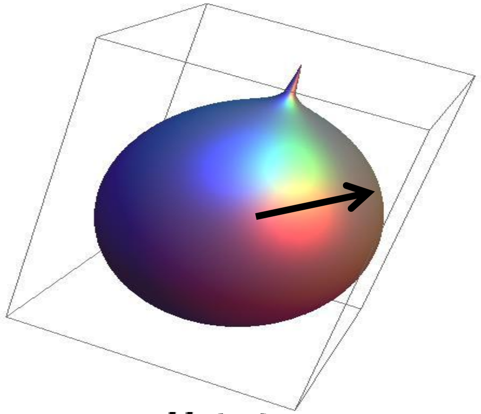
$$f'(R)R_{ab} - \frac{1}{2}f(R)G_{ab} - \nabla_a\nabla_b f' + G_{ab}\Box f' = 0.$$

+ boundary and initial conditions

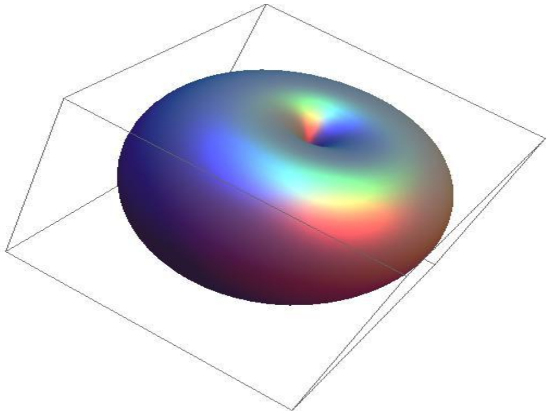
Geometry of sphere is well known solution for specific boundary conditions.

A form of extra space depends on boundary conditions

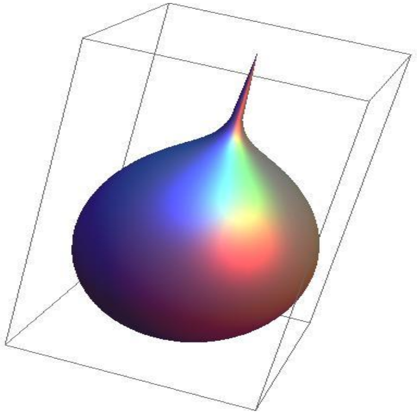
A set of stable solutions. Numerical results



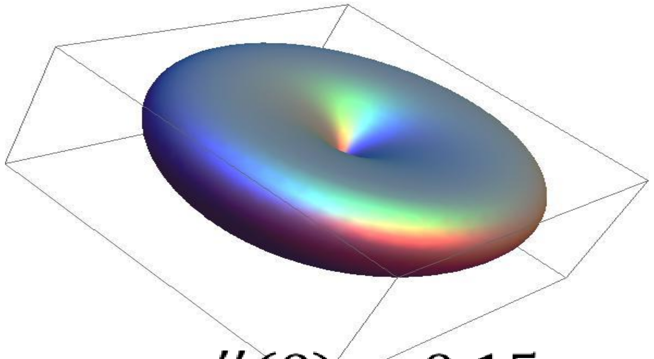
$a = r''(0) = -0.01$



$a = r''(0) = 0.05$



$a = r''(0) = -0.02$

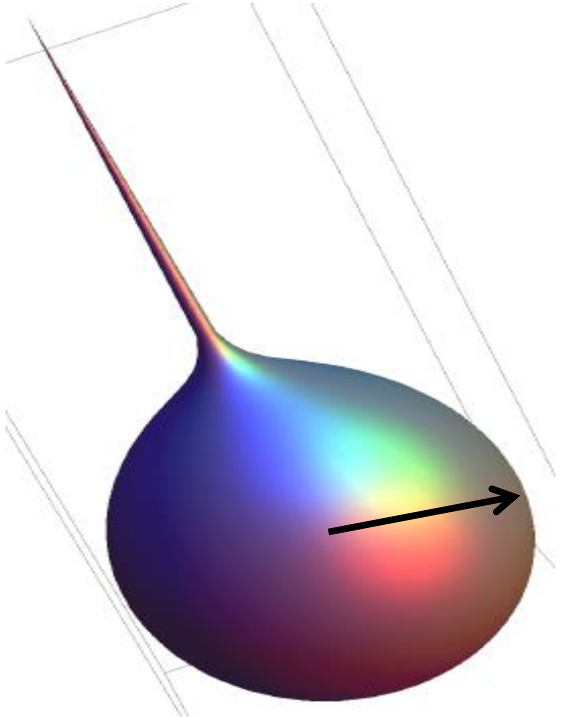


$a = r''(0) = 0.15$

Important!: Variety of boundary conditions – from space time foam

Asymptote

$$r(\theta) = \frac{C}{\sqrt{\theta}}, \quad \theta \to 0.$$



$$R(\theta) = \frac{5}{3C^2}\theta, \quad \theta \to 0$$

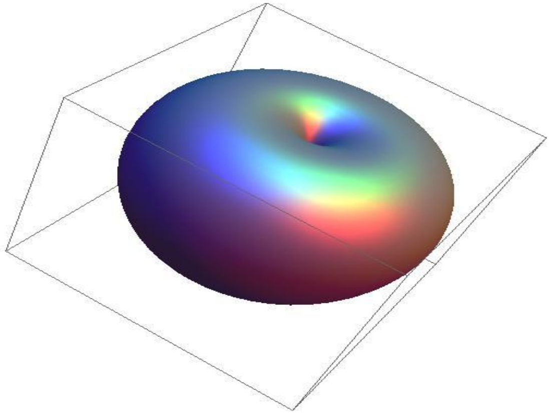
$$C = \left(\frac{3}{10}\left(\frac{U_2}{U_1} + R_0^2\right)\right)^{-1/4}$$

No boundary dependence!

Strange dark matter each particle has an individual mass!!

The case with dimensionality more than 2?
Detector interaction with a space volume filed by similar deformed extra space?

Field trapping by point like defects of 2-dim metric.
Brane formation.



$$L_m = \frac{1}{2}\partial_a\varphi G^{ab}\partial_b\varphi - \frac{m^2}{2}\varphi^2$$

The field is assumed to be uniformly distributed in our 4-dim space,

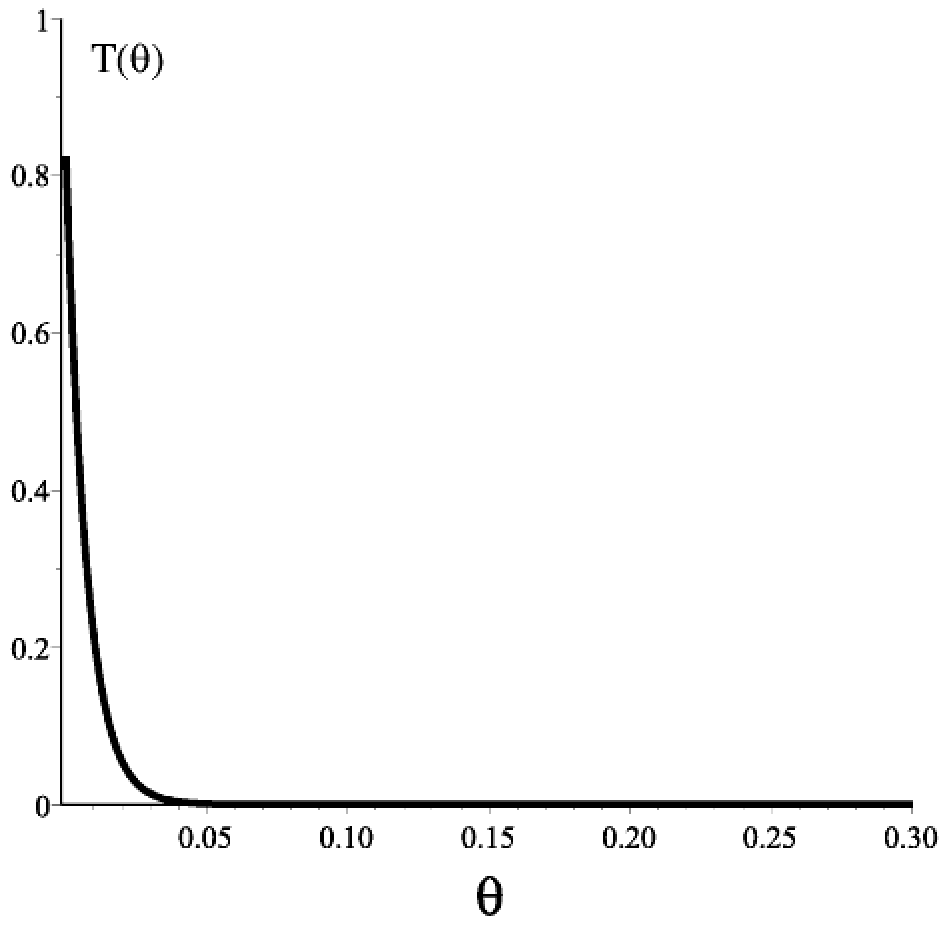
$$\varphi(x,y) = Y(y)$$

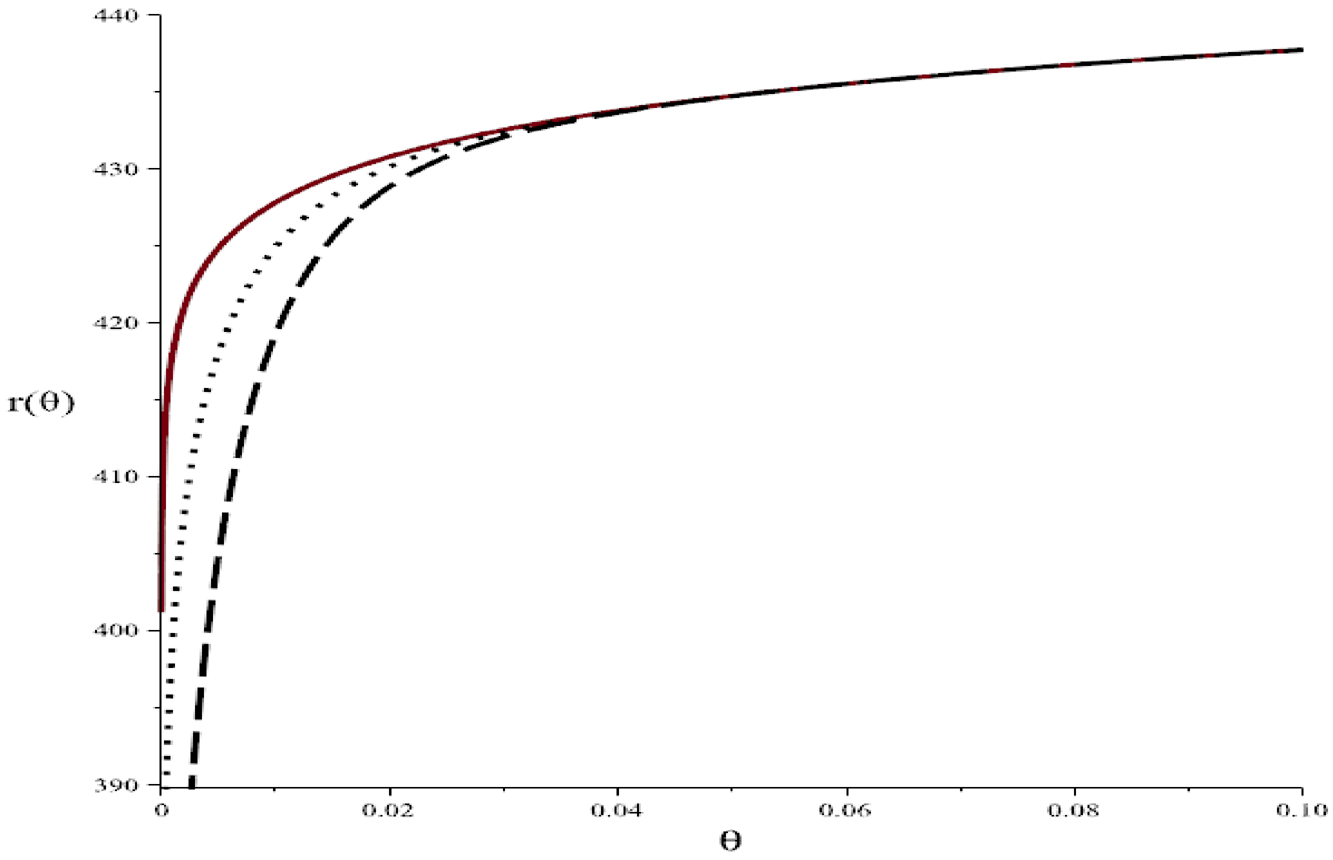
$$\cot(\theta)\partial_\theta Y(\theta) + \partial_\theta^2 Y(\theta) - m^2 r_b(\theta)^2 Y(\theta) = 0. \qquad (23)$$

In the WKB spirit a solution to (23) may be found in the form

$$Y(\theta) = Ce^{S/\epsilon}$$

$$Y(\theta) = C\exp\{-m\int_0^\theta d\theta' r_b(\theta')\}$$





The field is concentrated near singularity.
A brane is formed

Scalar field digs a gravitational well

Stability of the extra space deformation?
Are other particles trapped on the brane?

Why the cosmological term is small?

All known models suffer the fine tuning…

Anthropic principle???
Well, "we are living in such a universe because there are no life in universes with different properties".
It is supposed that a set of universes does exist and our universe belongs to this set.
Mechanism of creation of such set is needed.

Now we are able to do it. Remind:

Space time foam --- manifolds with any metrics --- variety of stationary metrics after evolution

Effective Lagrangian depends on extra space metric

Construction of universes set

$$S = \frac{m_D^4}{2} \int d^6 z \sqrt{-g_6} \left[f(R) + \frac{1}{2} \partial_A \varphi g_6^{AB} \partial_B \varphi - U(\varphi;\lambda) \right]$$

$$U(\varphi;\lambda) = \lambda_2 \varphi^2 + \lambda_3 \varphi^3 + \lambda_4 \varphi^4, \quad \lambda_3, \lambda_4 \ll \lambda_2 \sim 1$$

Units: $m_D = 1$

$$f(R) = u_1 (R - R_0)^2 + u_2$$

Let M is an energy scale where all parameters are defined, $v \ll M \ll r_c^{-1}$ (*)

Quantum corrections shift
coupling constants to
The maximal scale

$$\delta\lambda_i \sim \ln(m_D), m_D \sim 1 \rightarrow \quad \lambda_i \sim 1$$

Are we able to construct appropriate set (containing VERY small parameters) ?

Due to (*), extra space dynamics is faded out and we may take into account only classical configurations
Of the metric and scalar field

Classical equations depending on boundary conditions

$$\cot(\theta)\partial_\theta Y(\theta) + \partial_\theta^2 Y(\theta) - 2\lambda_2 r(\theta)^2 Y(\theta) = 0$$

$$Y(\pi) = Y_\pi; \quad Y'(\pi) = 0.$$

For 2-dim extra space:

$$\partial_\theta^2 R_n + \cot\theta \partial_\theta R_n = -\frac{1}{2} r(\theta)^2 \left[(R_0^2 - R_n^2) + \frac{u_2 + T}{u_1} \right].$$

As the additional conditions let us fix the metric at the point $\theta = \pi$

$$r(\pi) = r_\pi; \quad r'(\pi) = 0; \quad R(\pi) = R_\pi; \quad R'(\pi) = 0.$$

Both solutions $Y(\theta, Y_\pi), R_2(\theta, r_\pi)$ depend on specific boundary conditions.
The latter depend on initial conditions. Which are accidental.

After substitution classical solutions $Y(\theta, Y_\pi), R_2(\theta, r_\pi)$ to the action we obtain →

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{|g(x)|}(R_4 - 2\Lambda)$$

$$M_{Pl}^2 = 2\pi m_D^{D-2} \int d\theta \sqrt{|G(\theta)|} f'(R_2(\theta))$$

$$\Lambda \equiv \frac{\pi m_D^{D-2}}{M_{Pl}^2} \left[U_\phi(v_\phi) - \int d\theta \sqrt{|G(\theta)|} f(R_2) \right]$$

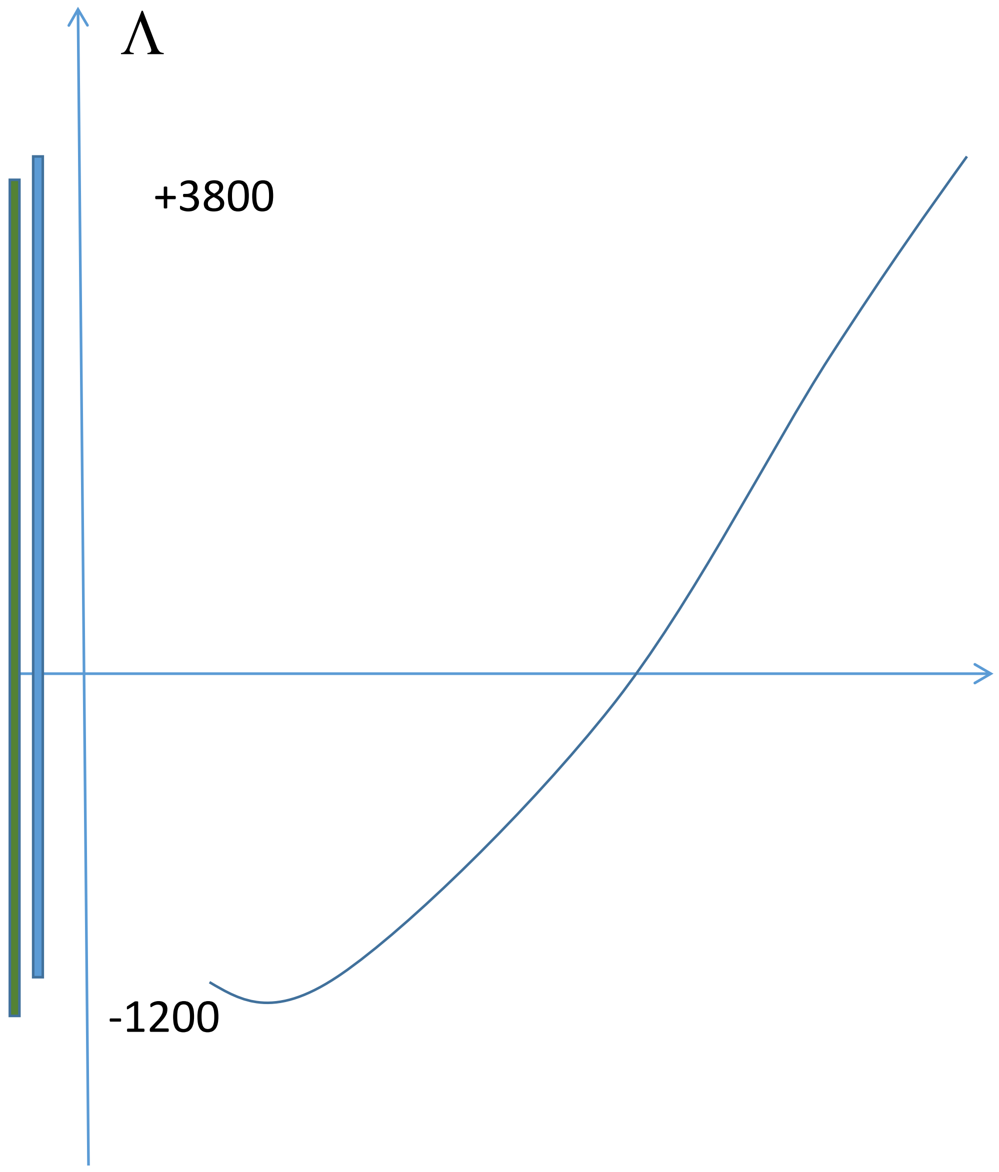
Everything depends on boundary (initial) conditions

Shifting down to low energies

$$\chi(x) = \frac{1}{\sqrt{K}} \chi_q(x) + \chi_s(x).$$

Integration out quick modes leads to small alternation of the interval

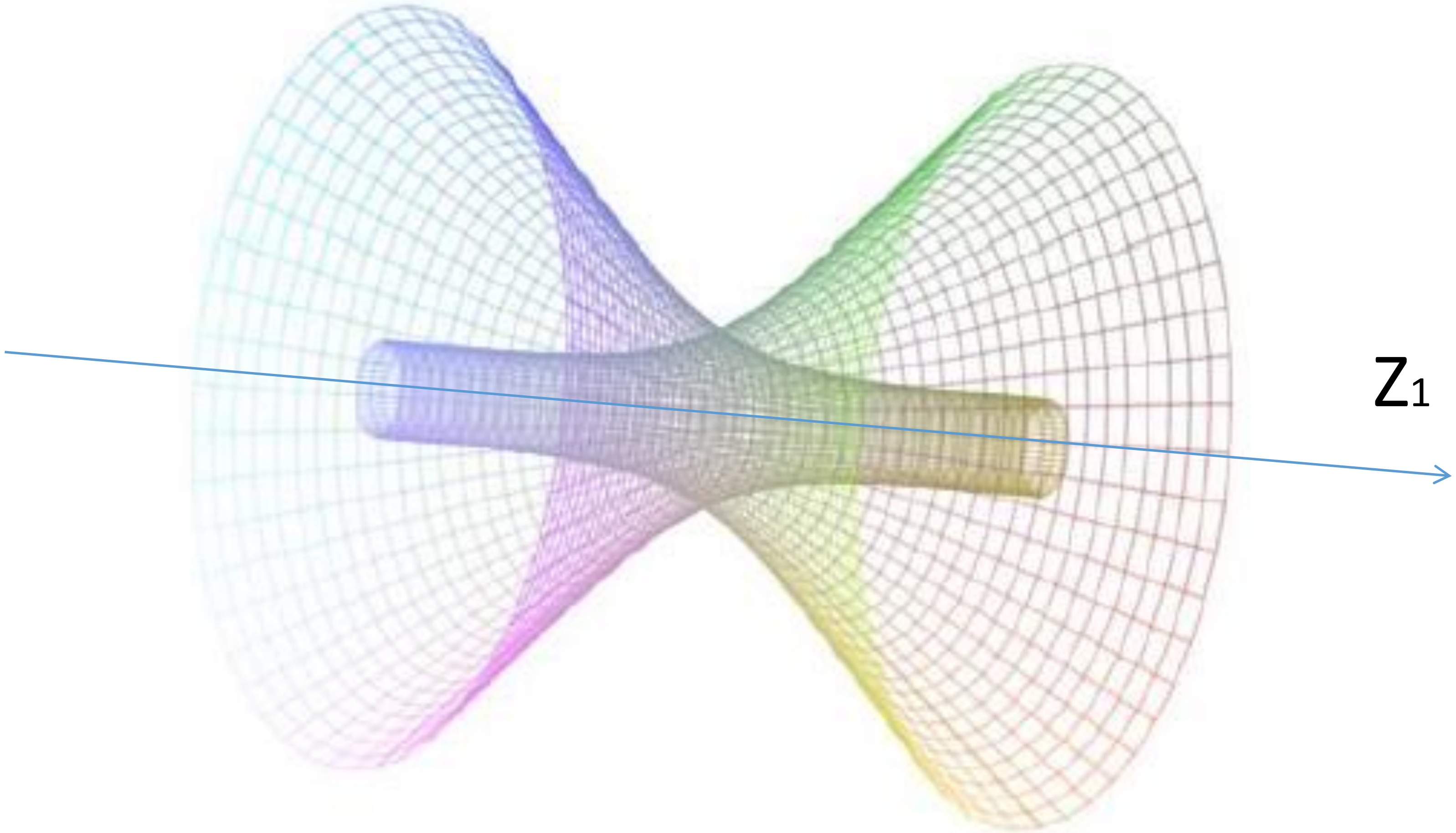
$$\delta\Lambda \simeq -\frac{1}{64\pi^2} M^4 \ln M^2 \quad << 1$$



Funnel to Extra Space

D-dim space exists before inflation starts.
D-4 dimensions are compact after inflation is finished.
The choice of 4 decompactified dimensions is accidental.

Consider the case of different large dimensions in contact


Z_1

Specific example

$$S = \frac{m_D^4}{2} \int d^6 Z \sqrt{|G|} \left[F(R) + c_1 R_{AB} R^{AB} \right]$$

$$F(R) = R + cR^2 - 2\Lambda$$

$$ds^2 = e^{2\alpha(u)} dt^2 - du^2 - e^{2\beta_1(u)} G_{1,ab} dy^a dy^b - e^{2\beta_2(u)} G_{2,mn} dz^m dz^n$$

$$R = R(\alpha, \beta_1, \beta_2) = \quad 2e_1^{-2\beta_1} + 2e^{-2\beta_2} - 2\alpha''(u) - 4\beta_1'' - 4\beta_2'' - 2\alpha'^2$$
$$-6\beta_1'^2 - 6\beta_2'^2 - 8\beta_1'\beta_2'$$

Boundary conditions

$$\alpha(u) = 0,\ \beta_1(u) = ln|u|, \beta_2(u) = \frac{1}{2} ln \left(\frac{2}{R_0} \right) \quad u \to +\infty,$$

$$\alpha = 0,\ \beta_2 = ln|u|, \beta_1 = \frac{1}{2} ln \left(\frac{2}{R_0} \right), \quad u \to -\infty,$$

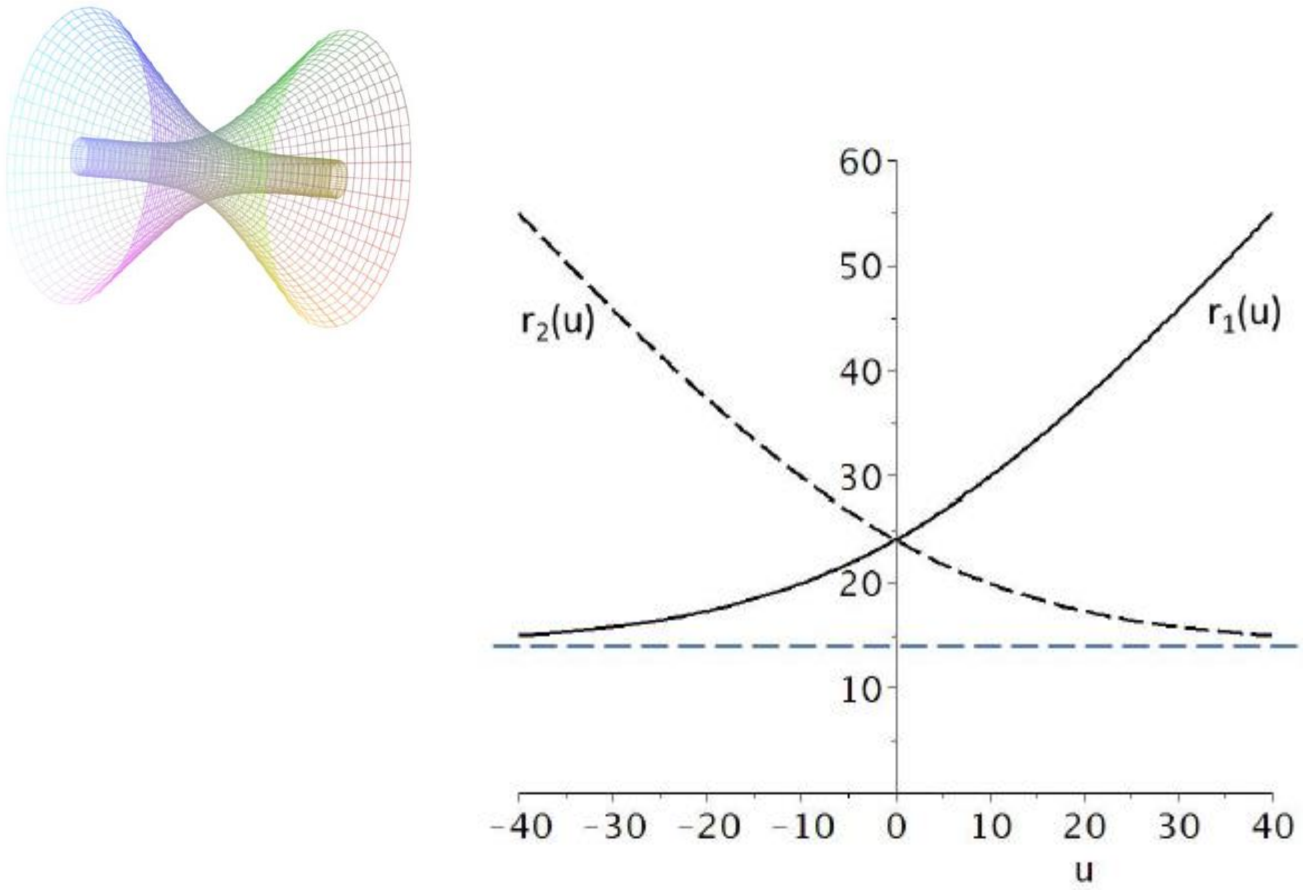


Figure 3: Two intersecting funnels. The size $r_{1,2}$ of the extra spaces $W_{1,2}$ vs. the parameter u. The horizontal line denotes the size r_{extra} of extra space at $u \to \pm\infty$. Parameter $w = 0.036$.

New stable 6-dim solution - two interpenetrating funnels –
looks like a wormhole (tunnel) from one extra space to another one

Scalar fields are not necessary

Nonlinear in the Ricci scalar gravity must be attracted

are there any solutions with MACROSCOPIC size of the funnel?

Limits to a size of compact extra space

Let some quantum fluctuation of the metric takes place in the region U with a characteristic size l and a volume l^D. It leads to a fluctuation of the Ricci scalar δR in a volume $\delta V_{D-4} \sim l^{D-4}$ of the extra space M_{D-4} and in a volume $\delta V_4 \sim l^4$ of the main space M_4. A deviation of the action from its classical value may be estimated as

$$\delta S \sim \frac{m_D^{D-2}}{2} \underbrace{\delta V_{D-4} \delta V_4}_{\sim l^D} \delta R. \qquad S = \frac{m_D^{D-2}}{2} \int d^D y \sqrt{G} R.$$

Quantum regime: $\delta S < 1.$ $\qquad \delta R \sim \bar{R}.$

$$l > l_{min} \equiv m_D^{\frac{2-D}{D}} \bar{R}^{-1/D} \qquad (*)$$

Estimations for a compact space as a whole: $l = L$

$$\bar{R} = n(n-1)/r_c^2$$

Positive curvature

$$L = r_c \quad \rightarrow \quad L > m_D^{-1}\,[n(n-1)]^{\frac{2}{D(2-D)}} \sim m_D^{-1}.$$

Negative curvature $V_{D-4} \simeq r_c^n e^{\alpha}$ $\quad \alpha = (n_{eff} - 1)L/r_c$ **The volume varies even if $r_c = const$**

Classical region: $r_c > r_{min} m_D^{-1}, \quad r_{min} = \left[\frac{(n_{eff}-1)^4}{n(n-1)}\alpha^{-4}e^{-\alpha}\right]^{1/(D-2)}.$

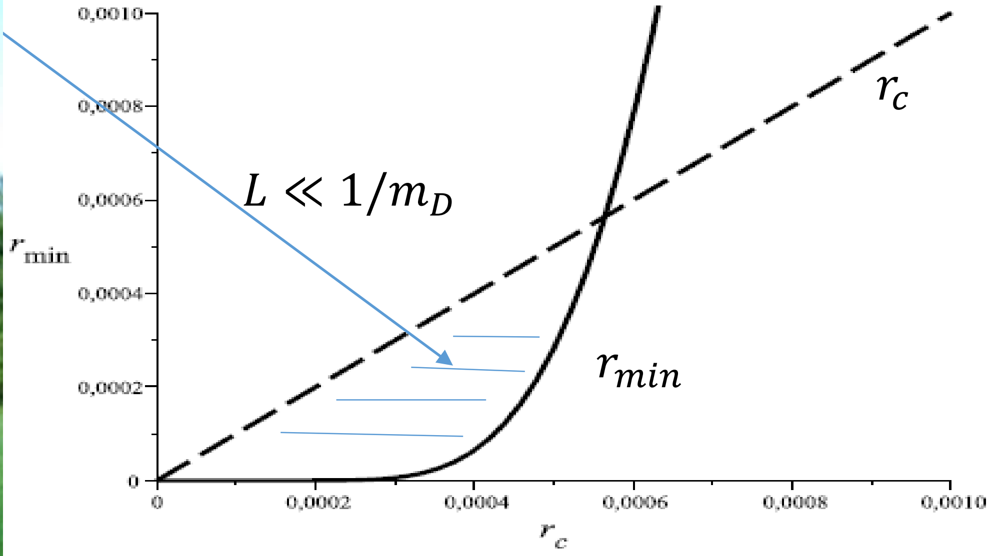


Figure 1: The curvature radius r_c (dashed line) must be larger than minimal curvature r_{min} (solid line) to suppress quantum effects. Numerical parameter values - $n = n_{eff} = 2$, the size of extra space $L = 0.01$ in m_D units.

Multidimensional gravity is able to be responsible for:

1. Inflation
2. Dark energy/matter.
3. Bosonic sector of Standard Model. The Higgs parameters are not the same as in the standard case
4. Baryon asymmetry of the Universe
5. Massive primordial BH
6. A set of universes with different properties
7. Time variation of physical parameters including *h* and G
8. Formation of observed symmetries

Future directions

1. Deformed extra space – dark matter, Hierarchi problem
2. Fermion involving
3. Funnels to extra space
4. Baryon asymmetry
5. String construction
6. Standard Model
7. What kind of extra space describes low energy physics as a whole?

Shortcomings

Renormalization.

The problem is not so serious if $m_D \sim 10\ TeV$:

Quantum corrections $\delta \sim 10^{19} \rightarrow 10^4$

Excess of fields.

Most of fields are heavy due to smallness of extra space

Stability of gravity with higher derivatives.

Does the Ostrogradsky theorem true for singular metrics?

СПАСИБО

Два пути развития

1. TOE (Theory Of Everything)

Наблюдаемая физика
Набор параметров
полей и симметрий

←

TOE
Ограниченный
набор параметров
полей и симметрий

←

?

Проблема тонкой настройки не решается

TOE (Theory Of Everything) – отодвигает решение проблем, но не решает их.

2. Multiverse
Дополнительные пространства + квантовые эффекты (Пространственно-временная пена)

Механизм формирования вселенных
с разными свойствами

Поля, число поколений

Внутренние симметрии

$$f_R(R)R_A^B - \frac{1}{2}f(R)\delta_A^B + \nabla_A\nabla^B f_R - \delta_A^B\Box f_R = 0,$$

Trial functions method

$$\alpha(u,w) = \frac{1}{2}\ln\left(\frac{1}{\cosh(wu)} + 1\right),$$

$$r_1(u;w) = \frac{1}{2}\int_{-\infty}^{u}(\tanh(wx)+1)dx + r_{extra}$$

$$r_2(u;w) = r_1(-u;w)$$

$$\alpha(u \to \pm\infty; w) = 0,$$

$$r_1(u \to +\infty; w) = u,\ r_1(u \to -\infty; w) = r_{extra},$$

$$r_2(u \to -\infty; w) = u,\ r_2(u \to +\infty; w) = r_{extra}$$

Search of action extrema by “w” variation

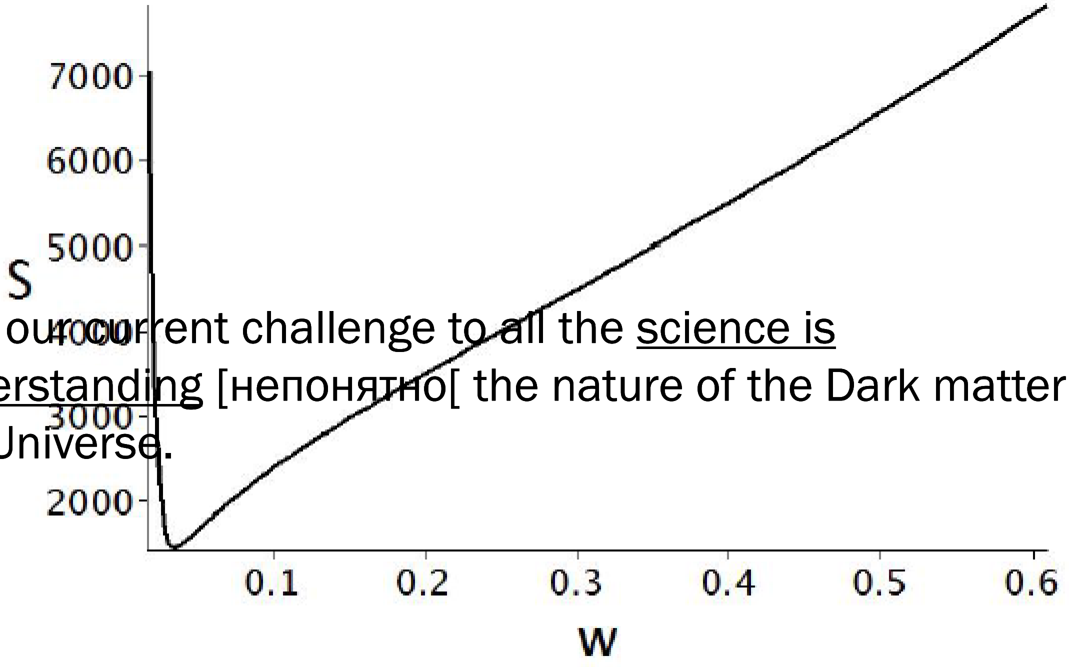


Figure 2: The action dependence on the parameter w with the minimum at $w = 0.036$. The Lagrangian parameters $v_1 = 5, v_2 = 0, R_0 = 0.01$.

Also our current challenge to all the science is understanding [непонятно[the nature of the Dark matter of the Universe.