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Solvability of the Boundary-Value Problem for a Partial Quasilinear Differential Equation of the Fourth Order

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Abstract—We use a topological method implying the reduction of the initial problem to solving an operational equation in a Hilbert space and consequent calculation of the rotation of the corresponding vector field. We show that in a sphere of a sufficiently large radius the problem has at least one generalized solution.

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1. INTRODUCTION. PROBLEM DEFINITION. CONSTRUCTING A FUNCTIONAL SPACE. THE NOTION OF A GENERALIZED SOLUTION TO THE PROBLEM

The voluminous literature is devoted to the study of boundary-value problems for nonlinear and quasilinear partial differential equations (for example, [1–7]). In this paper we study the nonlocal solvability of the first boundary-value problem for one class of quasilinear differential equations of the fourth order in a Sobolev space. Note that the operator in the left-hand side of the considered equation, generally speaking, is neither monotone nor an operator of semibounded variation, and the results adduced in this paper cannot be obtained as a particular case from solvability conditions for more general nonlinear problems studied in [1–7].

In a domain $\Omega \subset R^n$ with the boundary Γ we consider a quasilinear partial differential equation of the fourth order in the form

$$Lu \equiv \sum_{i,j,k,l=1}^{n} \frac{\partial^2}{\partial x_j \partial x_i}\left(a_{jikl}\frac{\partial^2 u}{\partial x_k \partial x_l}\right) + \sum_{i,j,l=1}^{n} b_{ijl}\frac{\partial^3 u}{\partial x_i \partial x_j \partial x_l} + \sum_{i,j=1}^{n} h_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} p_i \frac{\partial u}{\partial x_i} + qu + g(u_{x_k x_l}, \dots, u) = f, \quad (1)$$

where $u = u(x)$ is the desired function and the nonlinearity $g(u_{x_k x_l}, \dots, u)$ satisfies the conditions

$$\int_\Omega g(u_{x_k x_l}, \dots, u)u\,dx \geq c_0 \int_\Omega w^{2\nu}\,dx - c_1 \int_\Omega |d_1 u_{x_1} + \dots + d_n u_{x_n} + d_{n+1}u|\,|w|^\nu\,dx - c_2 \int_\Omega \widetilde{w}\,dx, \quad (2)$$

$$\left|\int_\Omega [g(u^1_{x_k x_l}, \dots, u^1) - g(u^2_{x_k x_l}, \dots, u^2)]v\,dx\right|$$

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