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GENERATORS AND DEFINING RELATIONS OF THE GENERALIZED FULL LINEAR GROUP OVER SEMILOCAL RINGS WITHOUT UNITY. II

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This paper is the final part of the studies we started in [1], where the generators and relations of the generalized full linear group $GL^\circ(n,\Lambda)$, $n \geq 2$, over an arbitrary semilocal ring Λ were found. In this paper, we prove the completeness of the system of relations 1–6 with respect to alphabet (5) from [1]. The enumerations of sections and formulas in this paper are the continuations of those in [1]. The first three sections are preparatory to the proof of the main result.

5. Basic lemmas

On the set of all words of alphabet (5), we introduce the relations $\overset{i}{\tilde{\to}}$, $1 \leq i < mn$, as follows: $W \overset{i}{\tilde{\to}} V$ if and only if these words are related by $W = X \circ V$, where X is a word which contains no nondiagonal expressions of the form (rj), $t_{rj}(\alpha)$, $r < j$, such that $r \leq i$. The relations $\overset{i}{\tilde{\to}}$ are reflexive and transitive. Let, for an index i, $1 \leq i < mn$, Q_i denote the set of all numbers t from $I(mn)$ such that $i \neq t \sim i$. Consider the forms $G_i = \prod\limits_{q\in Q_i} t_{iq}(\alpha_q)$ and $F_i = \prod\limits_{q\in P_i} t_{iq}(\alpha_q)$, where the order of factors is not significant (recall that $P_i = \{t \in I(mn) : i < t \sim i\}$). The fact that a letter $t_{ik}(\alpha_k)$ in G_i and F_i is zero will be indicated as follows: $G_i(\neq k)$ and $F_i(\neq k)$. The writings $G_i(\neq k, r)$, $F_i(\neq k, r)$, etc. have a similar meaning. We will say that a form $\widetilde{G}_i = \prod\limits_{q\in Q_i} t_{iq}(\beta_q)$ is *≡-contained* in a form G_i if $\beta_q \equiv \alpha_q$ and $\alpha_q = 0 \to \beta_q = 0$ for all $q \in Q_i$ (this agreement includes the notion of a form $\widetilde{F}_i$ ≡-contained in F_i). In what follows $I(G_i)$ and $I(F_i)$ denote the lengths of the forms G_i and F_i (i.e., the numbers of their nonzero letters) respectively.

Lemma 1. *Let a form $G_i = t_{ik}(\alpha_k) \circ t_{ir}(\alpha_r) \circ \cdots \circ t_{ij}(\alpha_j)$ be given, where $I(G_i) \geq 1$, and let all its letters be different from zero. Let $s, p, \dots, q$ be an arbitrary permutation of $k, r, \dots, j$. Then, applying relations* 2, 3a), 3c), 3d), *and* 5, *one can perform the transformation*

$$G_i \overset{i}{\tilde{\to}} t_{is}(\widehat{\alpha_s}) \circ t_{ip}(\widehat{\alpha_p}) \circ \cdots \circ t_{iq}(\widehat{\alpha_q}) = \widetilde{G}_i$$

(*i. e., permuting arbitrarily letters from G_i, we obtain a ≡-contained form*).

Proof follows from the relations of (structure) permutability

$$t_{ia}(\alpha) \circ t_{ib}(\beta) = t_{ib}(\widehat{\beta}) \circ t_{ia}(\widehat{\alpha}), \tag{18}$$

which are consequences of the relations mentioned in the statement of the lemma. In fact, if each of the letters $t_{ia}(\alpha)$, $t_{ib}(\beta)$ is diagonal, then $\alpha \equiv \beta \equiv 0$, and, according to relation 5, they can be permuted as follows: $t_{ia}(\alpha) \circ t_{ib}(\beta) = t_{ib}(\alpha) \circ t_{ia}(\beta)$. If only one of these letters is diagonal,