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Exact Estimate for the Rate of Decrease of a Solution to a Parabolic Equation of the 2nd Kind for $t \to \infty$

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INTRODUCTION

Let Ω be an arbitrary unbounded domain of the half-space $\mathbb{R}^n_+$, $n \geq 2$, $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$, $x_1 > 0$. In a cylindrical domain $D = \{t > 0\} \times \Omega$ we consider a linear parabolic equation of the second order

$$u_t = \sum_{i,j=1}^{n} (a_{ij}(t,x) u_{x_i})_{x_j}. \tag{1}$$

The coefficients $a_{ij}(t,x)$ of this equation are measurable functions, satisfying the condition of the uniform ellipticity with constants γ, Γ.

In this paper we study the rate of decrease for $t \to \infty$ of a solution to the first mixed problem for equation (1) with the initial boundary value conditions

$$u(t,x)|_{x \in \partial\Omega} = 0, \tag{2}$$

$$u(0,x) = \varphi(x), \quad \varphi(x) \in L_2(\Omega). \tag{3}$$

The dependence of the rate of decrease of solutions to mixed problems for a parabolic equation on geometric characteristics of an unbounded domain was first studied in papers [1]–[3]. In the mentioned papers under certain conditions of the isoperimetric type imposed on the domain one exactly estimates a solution to the second mixed problem

$$\sup_{x \in \Omega} |\mathbf{u}(t,x)| \leq C \|\varphi\|_{L_1(\Omega)} / v(\sqrt{t}), \quad v(r) = \operatorname{mes}\{x \in \Omega \,\big|\, |x| < r\}.$$

These investigations were continued in papers [4], [5] for the second mixed problem, and in [6], [7] for the first mixed problem. Some results were also obtained for parabolic equations of high order in [8], [9].

Papers [6], [7] are devoted to the rate of decrease of a solution to the first mixed problem (1)–(2). Determining the upper estimate and proving its exactness, in the mentioned papers one sets several technical requirements. In particular, in [7] for tubular domains

$$\Omega(f) = \{x \in R^n,\ x = (x_1, x') \,\big|\, |x'| < f(x_1)\}$$

these conditions are formulated as follows:

$$\lim_{r\to\infty} f(r) = \infty, \quad \lim_{r\to\infty} r/f(r) = \infty, \quad \lim_{r\to\infty} \frac{1}{\ln r} \int_1^r \frac{ds}{f(s)} = \infty. \tag{4}$$

One also assumes that a constant $A > 0$ exists such that for all sufficiently distant points $(z, 0)$ of the axis O_{x_1} the following inequalities are true:

$$A \int_{z-r/2}^{z+r/2} \frac{ds}{f(s)} \geq 1, \quad r(z) = \operatorname{dist}(\partial\Omega, (z,0)), \quad z \geq R_0. \tag{5}$$

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