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A MULTIPARAMETER FAMILY OF SOLUTIONS OF THE INTEGRAL VOLTERRA EQUATION WITH A SINGULARITY IN A BANACH SPACE

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In a real Banach space E we fix the norm $\|\cdot\|_E$. This norm induces in the space $L(E)$ of all linear bounded operators on E the operator norm

$$\|A\|_{L(E)} = \sup_{\|x\|_E=1} \|Ax\|_E.$$

In the space $C([0,T],E)$, as usual, we define the norm by the formula

$$\|\Psi\|_{C([0,T],E)} = \max_{0\le x\le T} \|\Psi(x)\|_E.$$

At last, in the space C of all continuous in the norm of $L(E)$ on the triangle $0 \le t \le x \le T$ functions with values in $L(E)$ we introduce the norm

$$|\,\|Q\|\,|_C = \max_{0\le t\le x\le T} \|Q(x,t)\|_{L(E)}.$$

We consider the integral Volterra equation

$$x^{m+1}u(x) = \int_0^x \rho(x,t)K(x,t)u(t)dt \quad (0 \le x \le T) \tag{1}$$

in $L_1([0,T],E)$, where $K(x,t)$ $(0 \le t \le x \le T)$ is a given function with values in $L(E)$ which has continuous partial derivatives up to the order $N+m+1$ (N, m are natural numbers), inclusively. Moreover, all partial derivatives up to the order $m-1$ equal zero at the point $(0,0)$, but there exist partial derivatives of the m-th order which differ from zero at the point $(0,0)$; $u(x)$ is the desired summable function on $[0,T]$ with values in E ($u \in L_1([0,T],E)$); $\rho(x,t)$ is a scalar positive homogeneous of zero degree function such that $\varphi(s) = \rho(1,s)$ is summable on $[0,1]$. Note that equation (1) was considered in simpler situations in [1]–[10].

We represent $K(x,t)$ by the Taylor formula

$$K(x,t) = \sum_{\alpha+\beta=m}^{\alpha+\beta=N+m} K^{\alpha\beta}x^\alpha t^\beta + \sum_{\alpha+\beta=N+m+1} \widetilde{K}^{\alpha\beta}(x,t)x^\alpha t^\beta$$

and introduce the operator sheaf

$$Q_\lambda - I = \sum_{\alpha+\beta=m} K^{\alpha\beta} \int_0^1 \varphi(s)s^{\beta+\lambda-1}ds - I. \tag{2}$$

This sheaf makes sense for values λ which satisfy the inequality

$$\int_0^1 \varphi(s)s^{\lambda-1}ds < \infty. \tag{3}$$