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ON THE SPECTRUM OF A CLASS OF SCHRÖDINGER OPERATORS WITH FINITELY MANY GENERALIZED FUNCTIONS

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Let $C^k(a,b)$ be a linear space of scalar complex-valued functions which are k times continuously differentiable on (a,b), $L_2(a,b)$ be a linear space of scalar complex-valued functions on (a,b), which have square summable modules, n, m be in $\mathbb{N}$ and fixed, $x_0=-\infty$, and $x_{m+n+1}=+\infty$.

Let us consider a formal differential expression

$$(A_{m,n}f)(x)=-f''(x)+q(x)f(x)+\sum_{k=1}^{m}\alpha_k\delta(x-x_k)f(x)+\sum_{k=m+1}^{m+n}\beta_{k-m}\delta'(x-x_k)f(x),\quad -\infty<x<+\infty.$$

In this formula, α_k $(k=1,2,\dots,m=\overline{1,m})$, β_k $(k=\overline{1,n})$, x_k $(k=\overline{1,m+n})$ are real numbers, $q(x)$ is a scalar real-valued nonnegative function on $(-\infty,+\infty)$ such that

$$\int_{-\infty}^{+\infty}(1+x^2)q(x)dx<\infty.$$

Besides, $x_1<x_2<x_3<\dots<x_{m+n+1}<x_{m+n}$.

Consider the equation

$$(A_{m,n}y)(x)=\lambda y(x),\quad x\in(-\infty,+\infty),\tag{1}$$

where λ is a complex number.

Let us agree to assume that any function $y(x,\lambda)$ on $(-\infty,+\infty)$, satisfying the conditions

1) $y\in C^2(x_k,x_{k+1})$ for $x\in(x_k,x_{k+1})$ with $k=\overline{0,m+n}$,
2) $-y''(x)+q(x)y(x)=\lambda y(x)$ for $x\in(x_k,x_{k+1})$ with $k=\overline{0,m+n}$,
3) $y(x_k+0)=y(x_k-0)=y(x_k)$, $y'(x_k+0)-y'(x_k-0)=\alpha_k y(x_k)$ for $k=\overline{1,m}$,
4) $y'(x_k+0)=y'(x_k-0)\equiv y'(x_k)$, $y(x_k+0)-y(x_k-0)=\beta_{k-m}y'(x_k)$ for $k=\overline{m+1,m+n}$

is a solution of this equation.

In connection with important applications to problems of the Quantum Mechanics (see [1]) it is of interest to study the spectral characteristics of the operator $A_{m,n}$.

It is well-known (see [2]) that the equation

$$-y''(x)+q(x)y(x)=\lambda y(x),\quad x\in(-\infty,+\infty)\tag{2}$$

has two linear independent solutions $\varphi_1(x,\lambda)$, $\varphi_2(x,\lambda)$, any solution of the equation $y(x,\lambda)$ has the following representation

$$y(x,\lambda)=C_1\varphi_1(x,\lambda)+C_2\varphi_2(x,\lambda),$$