

## A NEW APPROACH FOR SOLVING THE HILBERT BOUNDARY VALUE PROBLEM FOR ANALYTIC FUNCTION IN A MULTIPLY CONNECTED DOMAIN

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We suggest a new approach for solving the Hilbert boundary value problem, based of a direct construction of a solution of the homogeneous Hilbert problem.

Let  $D$  be an  $(m + 1)$ -connected domain bounded by closed simple nonoverlapping Lyapunov curves  $L_0, L_1, \dots, L_m$ , situated in the plane of complex variable  $z = x + iy$ , among which  $L_0$  embraces the others. It is required to determine a function  $F(z) = u(z) + iv(z)$  analytic and univalent in the domain  $D$ , which is continuously continuable to its boundary  $L = \bigcup_{k=0}^m L_k$ , via the boundary value condition

$$\operatorname{Re}[(a(t) + ib(t))F(t)] = a(t)u(t) - b(t)v(t) = c(t), \quad (1)$$

where  $a(t)$ ,  $b(t)$ , and  $c(t)$  are given on  $L$  real-valued functions of a point  $t$  of the contour  $L$ , which satisfy the Hölder condition and  $a^2(t) + b^2(t) \neq 0$  everywhere on  $L$ .

On  $L$  we establish a positive direction of path-tracing with which the domain  $D$  remains on the left.

Let  $t_{j0}$  be a fixed point of the curve  $L_j$ . In what follows, for a function  $f(t)$  given on  $L_j$ , by  $f(t_{j0} + 0)$  and  $f(t_{j0} - 0)$  we shall mean the limits to which  $f(t)$  tends as the point  $t$  tends to  $t_{j0}$  in the negative and positive direction, respectively. We shall denote by  $s$  the length of the arc of the curve  $L_j$ , counted from the point  $t_{j0}$  in the positive direction.

The boundary value condition (1) can be written as follows:

$$\operatorname{Re}[e^{-i\nu(t)} F(t)] = \frac{c(t)}{|G(t)|}, \quad (2)$$

where  $G(t) = a(t) - ib(t)$ ,  $\nu(t) = \arg G(t)$  is a branch continuous everywhere on  $L$  with possible except for points  $t_{j0}$  in which  $\nu(t_{j0} - 0) - \nu(t_{j0} + 0) = 2\pi \frac{k_j}{2}$ ; besides,  $k_j/2$  is integer,  $j = \overline{0, m}$ .

We call the number  $k = \sum_{j=0}^m k_j$  the *index of the Hilbert problem* (2) in following Muskhelishvili (see [1], p. 144) (note that in [2], p. 385, the number  $k/2$  was called the index of this problem).

Consider the homogeneous problem (2), when  $c(t) \equiv 0$  on  $L$ , i. e., the problem

$$\operatorname{Re}[e^{-i\nu(t)} F(t)] = 0, \quad (3)$$

or

$$|F(t)| \operatorname{Re}[e^{-i\nu(t)} e^{i\phi(t)}] = 0, \quad (4)$$

where  $\phi(t) = \arg F(t)$ .

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