ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2014, Vol. 58, No. 5, pp. 63–66. © Allerton Press, Inc., 2014.
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Solution of a Singular Homogeneous Hilbert Boundary-Value Problem for Analytic Function in Multiply Connected Circular Domain

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Received March 15, 2013

Abstract—We offer a new approach for solving the homogeneous Riemann–Hilbert boundary-value problem for analytic function in multiply connected circular domains. The approach is based on determination of analytic function in terms of known boundary values of its argument in a special case.

DOI: 10.3103/S1066369X14050090

Keywords: *Riemann–Hilbert boundary-value problem, index of a problem, Schwarz operator.*

We apply an approach from [1] for solving homogeneous Hilbert boundary-value problem for analytic function in multiply connected circular domain for singular (in I. N. Vekua's terms, see [2], P. 258) case where an index (wind number) of the problem is non-negative and less than connectivity order of the domain minus one. In that case the known results on solvability of the problem under consideration are not complete (see, e.g., [3], pp. 406, 407).

In the present paper we use notation and formulas from [1].

Let D be $(m+1)$-connected circular domain bounded by circles $L_0, L_1, \dots, L_m$ without common points lying in the plane of complex variable $z = x + iy$. We consider that the circle L_0 contains the rest circles.

We seek analytic and single-valued in the domain D function $F(z) = u(z) + iv(z)$ analytically extendable on the boundary $L = \bigcup_{j=0}^{m} L_j$ and satisfying boundary value condition

$$\mathrm{Re}[(a(t) + ib(t))F(t)] = a(t)u(t) - b(t)v(t) = 0, \tag{1}$$

where $a(t)$ and $b(t)$ are given defined on L real-valued functions of the point t of contour L satisfying the Hölder condition (i.e., of class H on L) and the inequality $a^2(t) + b^2(t) \neq 0$ everywhere on L.

We define on L positive orientation such that the domain D is situated from the left of L. Let t_{j0} be a fixed point of the curve L_j. In what follows symbols $f(t_{j0}+0)$ and $f(t_{j0}-0)$ stand for limits of defined on L_j function $f(t)$ for t tending to t_{j0} in negative and positive directions, correspondingly.

We rewrite the boundary value condition (1) as follows:

$$\mathrm{Re}[e^{-i\nu(t)}F(t)] = 0; \tag{2}$$

here $G(t) = a(t) - ib(t)$ and the branch $\nu(t) = \arg G(t)$ is continuous on L except, maybe, the points t_{j0} where $\nu(t_{j,0}-0) - \nu(t_{j,0}+0) = 2\pi\frac{\varkappa_j}{2}$, and $\varkappa_j/2$ is integer number, $j = \overline{0, m}$.

According to N. I. Muskhelishvili ([4], P. 144), we call the value $\varkappa = \sum_{j=0}^{m} \varkappa_j$ the index of the Hilbert problem (2). Note that in the books [2] (P. 243) and [3] (P. 385) an index of this problem is defined as $\varkappa/2$.

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