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Plane Domains with Special Cone Condition

A. N. Anikiev[1*]
(Submitted by S.K. Vodop'yanov)

[1] *Petrozavodsk State University, pr. Lenina 33, Petrozavodsk, 185910 Russia*
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Abstract—In the paper we present domains in $\mathbb{C}$ with special cone condition.

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1. Introduction. The domains satisfying the cone condition are of importance in various fields of mathematics (see, e.g., [1], [2], [3], P. 1076; [4], [5]). A domain G satisfies (weak) *cone condition* if $p + V(e(p), H) \subset G$ for any $p \in G$, where $V(e(p), H)$ stands for a right circular cone with vertex at the origin, with fixed angle ε and height H, $0 < H \leq \infty$, and with depending on p vector direction $e(p)$ of the axis.

The concept of α-accessible domain, $\alpha \in [0, 1)$, is introduced and studied in [6] (see also [7]): *A domain $\Omega \subset \mathbb{R}^n$, $0 \in \Omega$ is called α-accessible if for any point $p \in \partial\Omega$ there exists value $r = r(p) > 0$ such that cone*

$$K_+(p, \alpha, r) = \big\{x \in \mathbb{R}^n : \big(x - p, p/\|p\|\big) \geq \|x - p\| \cos\big(\alpha\pi/2\big), \|x - p\| \leq r\big\}$$

lies in $\Omega' = \mathbb{R}^n \setminus \Omega$.

In particular, the authors of [6] prove that for $\alpha \in (0, 1)$ α-accessible domains are bounded and satisfy the cone condition with $e(p)=-p$. In other words, the symmetry axis of cone must be radial, and this condition essentially restricts the domain Ω.

In the present paper we describe domains satisfying the cone condition where the symmetry axis of cone does not lie on a ray passing through the origin and the vertex of the cone, but this ray intersects the cone. Thus, the restriction $e(p) = -p$ is not necessary.

Definition. A domain $\Omega \subset \mathbb{C}$, $0 \in \Omega$, is called (α, β)-accessible, $\alpha, \beta \in [0, 1)$, if for any point $p \in \partial\Omega$ there exists a value $r = r(p) > 0$ such that cone

$$K_+(p, \alpha, \beta, r) = \big\{z \in \mathbb{C} : -\beta\pi/2 \leq \arg(z - p) - \arg p \leq \alpha\pi/2,\ |z - p| \leq r\big\}$$

lies in $\Omega' = \mathbb{C} \setminus \Omega$.

We denote $\alpha_0 = \min(\alpha, \beta)$, $\beta_0 = \max(\alpha, \beta)$. Let us note that class of (α, β)-accessible domains is intermediate between classes of α_0- and β_0-accessible domains.

The aim of the present work is to refuse fulfilment of the restriction $e(p) = -p$, and investigate the case where the angle of inclination (we denote it ϕ) of the symmetry axis of the cone to the radial beam $\{pt : t > 0\}$ is constant.

It is of interest to find out how the properties of domains change under such a change in the slope. Even in the case of constant angle ϕ this problem is rather sophisticated for large meanings of ϕ ($\phi > \frac{\pi}{2}$). Here the technique of the work [6] is not applicable.

2. The main results.

*E-mail: anikiev_a@mail.ru.