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Optimal Control in a Model of the Motion of a Viscoelastic Medium with Objective Derivative

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Abstract—In this paper we consider the Jeffreys model of the motion of a viscoelastic incompressible medium with the Yaumann derivative. Within this model we study the optimal control problem for the right-hand sides of the initial boundary value problem. We prove the existence of the optimal strong solution.

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1. INTRODUCTION

In this paper we study the optimal control problem for the right-hand sides connected with the following model of motion of a viscoelastic incompressible medium with the Yaumann derivative:

$$\frac{\partial v}{\partial t} - \mu_1 \Delta v + \sum_{i=1}^{n} v^i \frac{\partial v}{\partial x_i} + \nabla p = \operatorname{Div} \tau + u, \tag{1}$$

$$\frac{\mathcal{D}_0 \tau}{\mathcal{D} t} + \frac{1}{\lambda_1}\tau = 2\mu_2 \mathcal{E}(v), \tag{2}$$

$$\tau|_{t=0} = \tau_0, \tag{3}$$

$$v|_{[0,T]\times\partial\Omega} = 0, \ \ v|_{t=0} = v_0, \ \ \operatorname{div} v = 0, \ \ \int_\Omega p(t,x)dx = 0, t \in [0,T]. \tag{4}$$

Here $\frac{\mathcal{D}_0\Theta}{\mathcal{D}t} = \frac{\partial\Theta}{\partial t} + \sum\limits_{i=1}^{n} v^i \frac{\partial\Theta}{\partial x_i} + \Theta\mathcal{W} - \mathcal{W}\Theta$ is the Yaumann derivative of the tensor $\Theta = (\Theta_{ij})_{j=1,\dots,n}^{i=1,\dots,n}$; $\Omega \subset \mathbb{R}^n$ is a bounded domain whose boundary belongs to the class C^∞, $n = 2,3$; $v(t,x) = (v^1(t,x),\dots,v^n(t,x))$ is the field of velocities; $p(t,x)$ is the pressure function of the medium; $\mathcal{E}(v) = (\mathcal{E}_{ij}(v))_{j=1,\dots,n}^{i=1,\dots,n}$ is the tensor of deformation rates, $\mathcal{E}_{ij}(v) = \frac{1}{2}(\frac{\partial v^i}{\partial x_j} + \frac{\partial v^j}{\partial x_i})$, $i,j = 1,\dots,n$; $\mathcal{W}(v) = (\mathcal{W}_{ij}(v))_{j=1,\dots,n}^{i=1,\dots,n}$ is the vorticity tensor, $\mathcal{W}_{ij}(v) = \frac{1}{2}(\frac{\partial v^i}{\partial x_j} - \frac{\partial v^j}{\partial x_i})$, $i,j = 1,\dots,n$; $\tau = (\tau_{ij})_{j=1,\dots,n}^{i=1,\dots,n}$ is the purely elastic component of the deviator of the stress tensor σ, i.e., $\tau = \sigma - 2\mu_1\mathcal{E}(v)$; λ_1, μ_1, and μ_2 are some positive constants (they have a physical sense); u is the external force applied to the liquid, it represents the control parameter. We assume that the medium dense is constant and equals one.

This model is applied for the description of the motion of viscoelastic media such as liquid solutions of polymers, bitumens, concrete, Earth crust (see [1]). At present the existence of global with respect to time strong solutions to the initial boundary-value problem in the Jeffreys model with the objective derivative is an unsolved problem even in the plane case. In this paper we consider the optimal control problem for this system. The presence of the functional allows us to prove the existence of a strong solution for both two- and three-dimensional cases. An analogous result was obtained earlier in paper [2] for the Navier–Stokes system.

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