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Estimation on Curves of Dirichlet Series with a Convex Growth Majorant

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Abstract—For a class of Dirichlet series defined by a certain convex growth majorant we establish conditions for a sequence of indices which provide the implementation of precise estimates for their increase and decrease on curves that tend to infinity in a special way.

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Consider the entire transcendental function

$$f(z)=\sum_{n=1}^{\infty}a_nz^{p_n}\quad(p_n\in\mathbb{N}).$$

In [1] G. Pólya proves that if the function f has a finite order and the density of the sequence $\{p_n\}$ equals zero, then for any curve γ that goes to infinity along an arbitrary path (for example, along a spiral) a sequence $\{\xi_n\}$ $(\xi_n\in\gamma)$ exists such that

$$\ln M(|\xi_n|;f)=(1+o(1))\ln|f(\xi_n)|,$$

where $M(r;f)=\max\limits_{|z|=r}|f(z)|$. In [2] one completely extends the result obtained by G. Pólya to the Dirichlet series

$$F(s)=\sum_{n=1}^{\infty}a_ne^{\lambda_ns}\quad(s=\sigma+it,\ 0<\lambda_n\uparrow\infty)$$

of a finite Ritt order.

The goal of this paper is to study the case when $\ln M(\sigma)\le\Phi(m\sigma)$ $(m\in\mathbb{N})$, where $M(\sigma)=\sup\limits_{|t|<\infty}|F(\sigma+it)|$ and Φ is a certain convex increasing function. For instance, one can consider $\Phi(\sigma)=\underbrace{\exp\exp\cdots\exp}_{k}\sigma$ as the function Φ. Note that with $k=1$ the function F has a finite Ritt order (this case is considered in [2]).

Let $\Lambda=\{\lambda_n\}$ $(0<\lambda_n\uparrow\infty)$ be a sequence with a finite upper density

$$D=\varlimsup_{n\to\infty}\frac{n}{\lambda_n}<\infty.\tag{1}$$

Then

$$Q(z)=\prod_{n=1}^{\infty}\left(1-\frac{z^2}{\lambda_n^2}\right)$$

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