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A Problem with Tricomi and Frankl Conditions on the Characteristic for a Class of Mixed-Type Equations

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Abstract—In this paper we prove the correctness of a problem with Tricomi and Frankl conditions on the characteristic for a certain class of mixed-type equations.

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1. THE TRICOMI–FRANKL (TF) PROBLEM

Let Ω be a bounded simply connected domain in the complex plane $z = x + iy$ with the boundary consisting of the normal curve $\sigma_0 : x^2 + 4(m+2)^{-2}y^{m+2} = 1$ with endpoints $A(-1,0)$ and $B(1,0)$ for $y > 0$, and of characteristics AC and BC of the equation

$$\operatorname{sign} y|y|^m u_{xx} + u_{yy} - (m/2y)\, u_y = 0, \tag{1}$$

where $m = \text{const} > 0$, for $y < 0$.

Denote by Ω^+ and Ω^- the parts of Ω lying in half-planes $y > 0$ and $y < 0$, respectively. Let A_1 and A_2 denote, respectively, the points of intersection of the characteristic AC and characteristics starting from points $E_1\,(-c,0)$ and $E_2(c,0)$, $c \in (0,1)$.

The values of the required function in the Tricomi problem ([1], P. 29) are given for all points of the characteristic AC. Here we study the correctness of a problem different from the Tricomi one. More exactly, in the problem under consideration, parts AA_1 and A_2C of AC are free from the local boundary (Tricomi) condition; instead, we have the Frankl condition [2–5] on parts AA_1 and A_2C of AC and on segments AE_1 and E_2B of the line of degeneration.

Problem TF. Find a function $u(x,y) \in (\overline{\Omega})$ in Ω satisfying the following conditions:

1. $u(x,y) \in C^2(\Omega^+)$.

2. $u(x,y) \in R_1$ is a generalized solution $\tau'(x), \nu(x) \in H$ to (1) in Ω^- ([6], P. 104); denotations $\tau(x)$ and $\nu(x)$ are introduced below).

3. On the interval of degeneration AB the condition of conjugation takes place

$$\lim_{y\to -0}(-y)^{-\frac{m}{2}}\frac{\partial u}{\partial y} = \lim_{y\to +0} y^{-\frac{m}{2}}\frac{\partial u}{\partial y},\ \ x \in I \setminus \{-c, c\}, \tag{2}$$

where $I = (-1,1)$ is an interval of the axis $y = 0$, and limits for $x = \pm 1$ and $x = \pm c$ may have singularities of order less than 1.

4. The following conditions are fulfilled:

$$u|_{\sigma_0} = \varphi(x),\ \ x \in [-1,1], \tag{3}$$

$$u|_{A_1A_2} = \psi(x),\ \ x \in [-(1+c)/2, -(1-c)/2], \tag{4}$$

$$u[\theta(x)] - u[\theta(-x)] = u(-c,0) + \rho(x),\ \ x \in [-1,-c], \tag{5}$$

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