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The Lagrange Trigonometric Interpolation Polynomial With the Minimal Norm Considered as an Operator From $C_{2\pi}$ to $C_{2\pi}$

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Abstract—In this paper we perform a comparative analysis of Lebesgue functions and constants of a family of Lagrange polynomials. We prove that if a polynomial from the family has the minimal norm in the space of square summable functions, then it also has the minimal norm as an operator which maps the space of continuous functions into itself.

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INTRODUCTION

It is well-known (for example, [1], P. 24) that in the case of an even number $2n$ ($n \in N$) of interpolation nodes there exists a family $\Phi_n = \{\Phi_n(x,t;\lambda)\}$ of polynomials of order n

$$\Phi_n(x,t;\lambda) = \frac{1}{n}\sum_{k=0}^{2n-1} x(t_k)D_n^*(t-t_k) + \lambda\sin(nt+\alpha)$$
$$= \frac{1}{\pi}\int_0^{2\pi} x(s)D_n^*(t-s)d\omega_{2n}(s) + \lambda\sin(nt+\alpha), \quad (1)$$

where each of polynomials coincides with the function $x = x(t) \in \widetilde{R}$ at pairwise inequivalent roots of the equation $\sin(nt+\alpha) = 0$, where the polynomial

$$\Phi_n(x,t;0) = \Phi_n^*(x,t) = \frac{1}{n}\sum_{k=0}^{2n-1} x(t_k)D_n^*(t-t_k) \quad (2)$$

has the following property: The integral of its square on its period is minimal;

$$D_n^*(u) = (1/2)\sin(nu)\cot(u/2)$$

is the modified Dirichlet kernel of order n; $\widetilde{R} = \widetilde{R}[0,2\pi]$ is the set of integrable in the Riemann sense (R-integrable) 2π-periodic functions; $\omega_{2n} = \omega_{2n}(t)$ is a stepwise function associated with zeros of $\sin(nt+\alpha)$; $\alpha, \lambda \in \mathrm{R}$.

In the case of an odd number $2n+1$ ($n \in N$) of equidistant nodes such interpolation polynomial

$$P_n(x,t) = \frac{2}{2n+1}\sum_{k=0}^{2n} x(t_k)D_n(t-t_k)$$
$$\left(D_n(u) = \frac{\sin(n+0.5)u}{2\sin 0.5u};\ t_k = \frac{2\pi k}{2n+1}\ (k = \overline{0,2n})\right) \quad (3)$$

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