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A MIXED PROBLEM WITH DEVIATION FROM A CHARACTERISTIC FOR THE LAVRENT'EV–BITSADZE EQUATION WITH A COMPLEX PARAMETER

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1. Problem definition and main results

Consider the equation

$$Lu = u_{xx} + \operatorname{sgn} y u_{yy} + \lambda u = 0 \tag{1}$$

in a domain D bounded in the half-plane $y > 0$ by the arc of the unit circumference $BK = \Gamma$ ($r = 1$, $0 \le \varphi \le \varphi_0$, $0 < \varphi_0 \le \pi$) and the segment AK ($\varphi = \varphi_0$, $0 < r < 1$), and bounded in the half-plane $y < 0$ by the bounded segment AC of the straight line $y = -k_0 x$ and the segment CB of the characteristic $x - y = 1$ of equation (1), where $A(0,0)$, $B(1,0)$, $C(\frac{1}{k_0+1}, -\frac{k_0}{k_0+1})$.

Problem TN. Find a function $u(x,y)$, satisfying the conditions

$$u(x,y) \in C(\overline{D}) \cap C^1(D \cup AK \cup \Gamma) \cap C^2(D_+ \cup D_-), \tag{2}$$

$$Lu(x,y) = 0, \quad (x,y) \in (D_+ \cup D_-), \tag{3}$$

$$\left.\frac{\partial u}{\partial N}\right|_{AK} = 0, \tag{4}$$

$$u(x,y) = 0, \quad (x,y) \in AC, \tag{5}$$

$$\left.\frac{\partial u}{\partial N}\right|_{\Gamma} = f(\varphi), \tag{6}$$

where $\partial u/\partial N$ is the normal derivative, $D_+ = D \cap \{y > 0\}$; $D_- = D \cap \{y < 0\}$.

The generalized Tricomi problem for equation (1) for $\lambda = 0$ is studied in [1]; the Tricomi–Neumann problem for this equation is solved in [2]. In [3] this problem is considered in another domain. In this paper, using the results of [1]–[3], with the help of the method of spectral analysis we solve the mixed problem with deviation from a characteristic for the Lavrent'ev–Bitsadze equation with a parameter for all $\lambda \ne \lambda_{nm}$; λ_{nm} are eigenvalues of the corresponding spectral problem.

2. Problem on eigenvalues

Consider the spectral problem which corresponds to the generalized problem (2)–(6), also called

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