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CERTAIN ASPECTS OF REPRESENTATION OF FUZZY OPERATORS IN THE FORM OF A RATIO OF TWO POLYNOMIALS

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Aggregation of fuzzy information in the decision systems which are designed as a part of expert systems is associated with the use of the fuzzy union and intersection operators; therefore, there are actual the studies of some useful in applications properties of the operators, their structure, as well as the relations between various representations. On solving these problems we could realize a purposeful choice of the operator, thus enhancing the adequacy of the corresponding model.

Among the fuzzy operators mentioned in the literature we may select a class of union and intersection operators representable as a ratio of polynomials or, in particular, as a polynomial. Such are the following operators:

$$F_m(x,y) = \max(x+y-1,0); \qquad G_m(x,y) = \min(x+y,1);$$
$$F_p(x,y) = xy; \qquad G_p(x,y) = x+y-xy;$$
$$F_0(x,y) = \frac{xy}{x+y-xy}, \quad F_0(0,0)=0; \qquad G_{-1}(x,y) = \frac{x+y-2xy}{1-xy}, \quad G_{-1}(1,1)=1;$$
$$F_\alpha(x,y) = \frac{xy}{\alpha+(1-\alpha)(x+y-xy)}; \qquad G_\beta(x,y) = \frac{(\beta-1)xy+x+y}{1+\beta xy}.$$

Here F_m, F_p, F_0, and F_α are intersection operators, G_m, G_p, G_{-1}, and G_β are union operators (see [1], p.221).

Fuzzy operators are generalized by representing them in the class of triangular norms and conorms, the intersection operations being defined in terms of T-norms, while the unions — in terms of S-conorms (see [2], p.30).

A triangular T-norm is an operation $T : [0,1]\times[0,1] \to [0,1]$, satisfying the commutativity, associativity, monotonicity, and boundedness conditions $T(0,0) = 0$, $T(1,x) = x$. Its conorm (the S-conorm) is determined by the relation $S(x,y) = 1 - T(1-x, 1-y)$ and satisfies the same conditions, but its boundedness takes the form $S(1,1) = 1$, $S(0,x) = x$.

Let

$$\widetilde{\Phi}(x,y) = \frac{a_0 + a_1x + a_2y + a_3xy}{b_0 + b_1x + b_2y + b_3xy} \tag{1}$$

realize a union or intersection operation of the fuzzy sets.