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ASYMPTOTIC ANALYSIS OF SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH POLYNOMIALLY PERIODIC COEFFICIENTS

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We consider a class of nonautonomous differential equations and systems of the latter with polynomially periodic coefficients of the form

$$\dot{x} = \Big(\sum_{k=0}^{\infty} A_k(t)t^{m-k}\Big)x, \quad x(t_0) = x^0, \quad t \geq t_0 > 1, \quad m \geq -1, \quad x \in R^n, \tag{1}$$

where $A_k(t)$ ($k \geq 0$) are sufficiently smooth and T-periodic on the semiaxis $[t_0, +\infty)$ matrix functions. The obtained results supplement those of [1]–[3]. One can obtain the mentioned systems (for constant A_k), reducing the hypergeometric equations

$$p(t)\ddot{x} + q(t)\dot{x} + \lambda x = 0,$$

where $q(t)$ and $p(t)$ are polynomials, whose degrees do not exceed one and two, correspondingly, λ is a constant parameter, their special cases, equations of Airy, Bessel, Hermite and many others.

For a square matrix A denote

$$A = \{a_{jk}\}_1^n, \quad \overline{A} = \operatorname{diag}\{a_{11}, \dots, a_{nn}\}, \quad \overline{\overline{A}} = A - \overline{A}.$$

Theorem 1. *The Cauchy problem* (1) *for* $m = -1$ *in the case when the matrix* A_0 *is constant and its spectrum* $\{\lambda_{0j}\}$ *satisfies the inequalities*

$$\sigma_{jk} \equiv \lambda_{0j} - \lambda_{0k} \neq 0, \pm 1, \pm 2, \dots, \quad \operatorname{Re}\lambda_{0j} \leq 0, \quad j \neq k, \quad j,k = \overline{1,n},$$

has a unique and bounded for $t \to +\infty$ *solution* $x(t)$, *representable in the form*

$$x(t) = S_0 H_{(N)}(t) \exp\Big(\int_{t_0}^{t} s^{-1}\Lambda_{(N)}(s)ds\Big)C + O(t^{-N-2}),$$

where $S_0^{-1}A_0S_0 = \Lambda_0 = \operatorname{diag}\{\lambda_{01}, \dots, \lambda_{0n}\}$, *and matrices*

$$H_{(N)}(t) = E + \sum_{k=1}^{N} \overline{\overline{H}}_k(t)t^{-k}, \quad \Lambda_{(N)} = \Lambda_0 + \sum_{k=1}^{N} \Lambda_k(t)t^{-k}$$

with T*-periodic coefficients are uniquely defined during the proof.*

The proof is based on the fact that for sufficiently large $t \gg 1$ the replacement $x = S_0H_{(N)}(t)z$ in system (1) results in a system with an almost diagonal matrix

$$t\dot{z} = \big(\Lambda_{(N)}(t) + t^{-N-1}R(t)\big)z \equiv Q(t)z, \quad z(t_0) = z^0, \quad \|R(t)\| \leq C, \quad t \geq t_0 > 0.$$