

# Multidimensional Triangle-Truncated Simplexes

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**Abstract**—We consider the convex polytopes, called triangle-truncated simplexes. From the point of view of a constructive object in four and higher dimensions vector space such polytopes are multidimensional analogs of one classical semi-regular polytopes, namely, truncated tetrahedron. We present results of investigations of inner geometrical structure and combinatorial characteristics of the complete assemblage of faces of triangle-truncated simplexes in vector spaces of arbitrary dimension. We formulate and prove a theorem about the volumes of multidimensional truncated simplex of generalized kind in Euclidean space.

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## INTRODUCTION

Study of certain types of the constructive fractal simplex-sets [1] in linear spaces of dimension greater than equal to 4 shows us strong relation between these fractals and the truncated simplexes which represent a multidimensional analog of one classical semi-regular polytope, namely the truncated tetrahedron.

In general many types of multidimensional truncated polytopes are now not only well-known but also thoroughly described [2–6]. At the same time in contrast with the three classical regular convex polytopes (multidimensional simplex, cube and cocube) whose combinatorial characteristics of the complete  $i$ -faces set,  $i = \overline{0, n-1}$  can be written down explicitly [2], [4], [6–11], the author has not yet seen similar relations for truncated polytopes, particularly, triangle-truncated simplexes in vector spaces of arbitrary dimension.<sup>1)</sup>

Since the multidimensional triangle-truncated simplexes turned out to be relatively nontrivial geometric objects with the complicated inner structure, their combinatorial characteristics study is a self-consistent problem.

In [1] the author considered the limit case of the simplexes truncation in vector spaces of arbitrary dimension (mid-truncated simplexes).

## 1. GEOMETRIC STRUCTURE AND COMBINATORIAL CHARACTERISTICS OF MULTIDIMENSIONAL TRIANGLE-TRUNCATED SIMPLEXES

Let us introduce the following definition of the multidimensional triangle-truncated simplex in a vector space of an arbitrary dimension.

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<sup>1)</sup>We know the numerical characteristics of the complete faces set only for certain types of the truncated polytopes in small dimensions.