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INFINITESIMAL BENDING OF TORUS-LIKE SURFACE WITH POLYGONAL MERIDIAN

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Introduction. In the present article we consider infinitesimal bending of a torus-like surface with polygonal meridian. We obtain necessary and sufficient conditions for this surface to be nonrigid. We also present a procedure of constructing a field of infinitesimal bending for this surface. Note that K.M. Belov's result (see [1]) is a partial case of the theorem we shall prove here.

Let a simple polygon p_n with the vertices $A_i(u_i, \rho_i)$ $(i = 1, \dots, n)$, given with respect to a Cartesian coordinate system $u0\rho$, rotate about an axis u, whose unit vector is $\bar{e}$. Then the side equation has the form

$$A_mA_{m+1}: \ \rho_{(m)} = \rho_m + \frac{\rho_{m+1} - \rho_m}{u_{m+1} - u_m}(u - u_m), \quad \rho'_{(m)} = \frac{\rho_{m+1} - \rho_m}{u_{m+1} - u_m} = k_m \qquad (1)$$
$$(m = 1, \dots, n; \ A_{n+1} \equiv A_1),$$

where $\rho_{(m)}$ is the value of ρ on A_mA_{m+1}.

In order to consider an infinitesimal bending of a torus-like surface of revolution with a simple closed meridian we use the Cohn-Vossen method (see [2]).

A radius-vector of surface point can be written as

$$\bar{r}(u, v) = u\bar{e} + \rho(u)\bar{a}(v), \qquad (2)$$

where $\rho = \rho(u)$ is the meridian equation, $\bar{a}(v)$ is the unit vector of the axis ρ, v is the angle between the initial meridian plane and $\bar{a}(v)$, and $\bar{e}$ is the unit vector of the axis of revolution.

The fundamental field of infinitesimal bending of the surface (2) is

$$\bar{z}(u, v) = [\varphi_k(u)e^{ikv} + \tilde{\varphi}_k e^{-ikv}]\bar{e} + [\psi_k(u)e^{ikv} + \tilde{\psi}_k(u)e^{-ikv}]\bar{a}(v) + \\ + [\chi_k(u)e^{ikv} + \chi_k(u)e^{-ikv}]\bar{a}'(v), \qquad (3)$$

where, for example, $\tilde{\varphi}_k(u)$ is the conjugate value of $\varphi_k(u)$, and the functions $\varphi_k(u)$, $\psi_k(u)$, $\chi_k(u)$ satisfy the equations

$$\begin{aligned} &\varphi'_k(u) - \rho'(u)\psi'_k(u) = 0, \quad \psi_k(u) + ik\chi_k(u) = 0, \\ &ik\varphi_k(u) + \rho'(u)[ik\psi_k(u) - \chi_k(u)] + \rho(u)\chi'_k(u) = 0. \end{aligned} \qquad (4)$$

The functions $\psi_k(u)$, $\chi_k(u)$ satisfy also the equation

$$\rho(u)\lambda''(u) + (k^2 - 1)\rho''(u)\lambda(u) = 0, \qquad (5)$$

where $\lambda(u)$ is the unknown function. Omitting the index k, we denote by $\psi_{(i)}$ the value of ψ on the sides A_iA_{i+1}, $i = 1, \dots, n$, and $A_{n+1} \equiv A_1$.

From (1), (5) it follows that $\psi_{(i)}(u)$ are linear functions, i. e.,

$$\psi_{(i)}(u) = M_iu + N_i \quad (i = 1, \dots, n). \qquad (6)$$