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A THEOREM ON α-DISTRIBUTION FOR NORMAL MANIFOLDS OF THE KILLING TYPE

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Presently, the geometry of almost contact metric manifolds ($A\mathfrak{C}$-manifolds) is actively studied. The $A\mathfrak{C}$-manifolds endowed with additional structures are of a significant interest for differential-geometric investigations. For example, the $A\mathfrak{C}$-manifolds endowed with normal structures of the Killing type ($\mathcal{CNK}$-manifolds) are generalizations of the Sasakian, quasi-Sasakian, cosymplectic, and exactly cosymplectic manifolds. Those $\mathcal{CNK}$-manifolds, which are locally conformal to almost contact metric manifolds with closed fundamental 2-form ($\mathcal{LC}c$-manifolds), are of a particular interest. For this case we obtain a contact analog of the locally conformal Kähler manifolds (see, e. g., [1]–[4]), which find applications in modern Mathematical Physics (see [5]).

In this article we study the properties of an involutive distribution contained in the α-distribution of an $\mathcal{LC}c$-manifold. The notion of an α-distribution (Kähler distribution) for the locally conformal Kähler manifolds was introduced by K. Ikuta in [6]. Studying an involutive distribution, which is contained in the Kähler distribution under condition that the Lee form of the manifold is covariantly constant (i. e., when the manifold is a generalized Hopf manifold, see [7]), he established that 1) under the condition that such a distribution is anti-invariant, its dimension can be at most $m-1$ (m is the complex dimension of the manifold), 2) each involutive distribution contained in the Kähler distribution is anti-invariant. Later this problem was studied by N.N. Shchipkova for the Vaisman–Gray manifolds. For these manifolds a generalization for the last of the above results by K. Ikuta was obtained. The theorem proved in this article is both a contact analog and a generalization of the results obtained by K. Ikuta and N.N. Shchipkova. We also prove that, if the Lee form of an $\mathcal{LC}c$-manifold is covariantly constant, then any involutive distribution contained in its α-distribution is anti-invariant.

Let M be a smooth manifold endowed with an $A\mathfrak{C}$-structure, i. e., a collection $\{\eta, \xi, \Phi, g = \langle\ ,\ \rangle\}$ of tensor fields on M, where ξ is a vector, η a covector, Φ an endomorphism of the module $\mathfrak{X}(M)$ of smooth vector fields on M, g a Riemannian metric, related to each other as follows:

$$\eta(\xi) = 1,\quad \Phi(\xi) = 0,\quad \Phi^2 = -\,\mathrm{id} + \xi\otimes\eta,\quad \eta\circ\Phi = 0,$$
$$\langle \Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta(X)\eta(Y),\quad X, Y \in \mathfrak{X}(M).$$

Let ∇ be the Riemannian connection on M, d the operator of the exterior differentiation, δ the operator of the codifferentiation, $C^\infty(M)$ the algebra of smooth functions on M, $\Omega(X,Y) = \langle X, \Phi Y\rangle$ the fundamental 2-form of the structure, and $\Theta = \frac{1}{n-1}\delta\Omega\circ\Phi$ the Lee form. In what follows we assume that the Lee form Θ is nonzero at each point of M. A manifold admitting an $A\mathfrak{C}$-structure is called an $A\mathfrak{C}$-manifold. It is well-known (see [8]) that an $A\mathfrak{C}$-manifold is odd-dimensional and orientable. The formulas $\mathfrak{m} = \xi\otimes\eta$, $\mathfrak{l} = \mathrm{id} - \xi\otimes\eta$ determine the two complementary projectors onto the $C^\infty(M)$-module $\mathfrak{X}(M)$ of an $A\mathfrak{C}$-manifold M. Let $\operatorname{Im}\mathfrak{m} = \mathfrak{M}$, $\operatorname{Im}\mathfrak{l} = \mathfrak{L}$, then $\mathfrak{X}(M) = \mathfrak{M}\oplus\mathfrak{L}$.
