ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 3, pp. 24–30. © Allerton Press, Inc., 2010.
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A Nonlocal Problem for the Bitsadze–Lykov Equation

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Received November 22, 2007

Abstract—We study a nonlocal boundary-value problem for a degenerate hyperbolic equation. We prove that this problem is uniquely solvable if Volterra integral equations of the second kind are solvable with various values of parameters and a generalized fractional integro-differential operator with a hypergeometric Gaussian function in the kernel.

DOI: 10.3103/S1066369X10030059

Key words and phrases: *boundary-value problem, fractional integro-differential operator, Volterra integral equation.*

Introduction. Consider the equation

$$y^2u_{xx} - u_{yy} + au_x = 0, \;\; |a| < 1, \tag{1}$$

which is said to be the Bitsadze–Lykov equation or the moisture transfer equation ([1], P. 234), in a domain D bounded by characteristics $AC = \{(x,y) : x - \frac{y^2}{2} = 0, \, y \le 0\}$, $BC = \{(x,y) : x + \frac{y^2}{2} = 1, \, y \le 0\}$, and the segment $AB = J = \{(x,y) : 0 \le x \le 1, y = 0\}$.

Following papers [2, 3] (see also [4], pp. 326–327), we introduce generalized fractional integrals and derivatives with the hypergeometric Gaussian function ${}_2F_1(a,b;c;z)$:

$$\begin{gathered}(I_{0+}^{\alpha,\beta,\eta}\varphi)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}F\left(\alpha+\beta, -\eta; \alpha; 1-\frac{t}{x}\right)\varphi(t)dt,\\ \alpha>0, \;\; 0<x<1,\\ (I_{0+}^{\alpha,\beta,\eta}\varphi)(x) = \left(\frac{d}{dx}\right)^n (I_{0+}^{\alpha+n,\beta-n,\eta-n}\varphi)(x),\\ \alpha<0, \;\; 0<x<1, \;\; n=[-\alpha]+1,\end{gathered} \tag{2}$$

$$\begin{gathered}(I_{1-}^{\alpha,\beta,\eta}\varphi)(x) = \frac{(1-x)^{-\alpha-\beta}}{\Gamma(\alpha)}\int_x^1 (t-x)^{\alpha-1}F\left(\alpha+\beta, -\eta; \alpha; \frac{t-x}{1-x}\right)\varphi(t)dt,\\ \alpha>0, 0<x<1,\\ (I_{1-}^{\alpha,\beta,\eta}\varphi)(x) = \left(-\frac{d}{dx}\right)^n (I_{1-}^{\alpha+n,\beta-n,\eta-n}\varphi)(x),\\ \alpha<0, \;\; 0<x<1, \;\; n=[-\alpha]+1;\end{gathered} \tag{3}$$

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