ISSN 1066-369X, Russian Mathematics , 2016, Vol. 60, No. 10, pp. 5–9. © Allerton Press, Inc., 2016.
Original Russian Text © L.E. Evtushik, I. Hinterleitner, N.I. Guseva, J. Mikeš, 2016, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2016, No. 10, pp. 8–13.

Conformal Mappings onto Einstein Spaces

L. E. Evtushik[†1], I. Hinterleitner[2*], N. I. Guseva[3], and J. Mikeš[4***]**

[1] *Moscow State University,*
Vorob'yovy Gory 1, Moscow, 119991 Russia

[2] *Brno University of Technology, ul. Žižkova 17, Brno, 60200 Czech Republic*

[3] *Moscow State Pedagogical University*
ul. Malaya Pirogovskaya 1, Moscow, 119882 Russia

[4] *Palacky University*
ul. 17.listopadu 12, Olomouc, 77 146, Czech Republic

Received March 11, 2015

Abstract—We study conformal mappings of Riemannian manifolds onto Einstein manifolds under minimal condition on the differentiability class of manifolds in question. We establish under what conditions the linear equations obtained by J. Mikeš, M. L. Gavril'chenko and E. I. Gladyscheva that define such mappings.
We obtain an estimate of the number of essential parameters on which the general solution of the fundamental system of equations depend.

DOI: 10.3103/S1066369X16100029

Keywords: *(pseudo-)Riemannian space, conformal mapping, Einstein space.*

INTRODUCTION

Conformal mappings of n-dimensional Riemannian spaces V_n were studied by many researchers. We assume that the metrics g of V_n under study are of arbitrary signature, i.e., V_n is either a proper Riemannian space or a pseudo-Riemannian space. Conformal mappings have applications in the general theory of relativity (see, e. g., [1–5]).

The question on whether V_n $(n > 2)$ admits a conformal mapping onto an Einstein space $\overline{V}_n$ was reduced by H. Brinkmann [6] to the problem of existence of a solution to some nonlinear system of differential equations in covariant derivatives of the Cauchy type with respect to $(n + 1)$ unknown functions. This problem is stated in detail in A. Z. Petrov's monograph [2].

In [7], and also in [8, 9], [10] (pp. 112–116), the basic system of this problem was reduced to a linear system with the use of which the degree p of parametrical arbitrariness in the solution of the indicated problem was estimated (in our terminology, the *degree of mobility of a Riemannian space with respect to conformal mappings onto Einstein spaces*).

In [11] the first lacuna in the distribution of mobility degrees of Riemannian spaces with respect to conformal mappings onto Einstein spaces was estimated. As is known [7], conformally flat Riemannian spaces and only they admit the maximum value $p = n + 2$. We find a tensor criterion of spaces different from conformally flat ones with $p = n - 1$, which is the maximum possible value. Thus, we have obtained the estimate for the first lacuna in the distribution of mobility degrees of Riemannian spaces with respect to conformal mappings onto Einstein spaces and singled out the maximal mobile spaces different from conformally flat with respect to the indicated degrees. Note that equations obtained in [7], rewritten in operator form, appear, e.g., in [12, 13].

[†] Deceased.
[*] E-mail: Hinterleitner.Irena@seznam.cz.
[**] E-mail: Ngus12@mail.ru.
[***] E-mail: Josef.Mikes@upol.cz.