ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2012, Vol. 56, No. 1, pp. 48–54. © Allerton Press, Inc., 2012.
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Application of the Riemann Method to a Factorized Equation in an n-Dimensional Space

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Received January 22, 2011

Abstract—We obtain sufficient conditions for the unique solvability of the characteristic boundary problem for one hyperbolic equation.

DOI: 10.3103/S1066369X12010070

Keywords and phrases: *Riemann method, hyperbolic equation.*

In papers [1–5] one studies some problems in triangular domains for the equation

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)\left(\frac{\partial^2 u}{\partial x\partial y}+a(x,y)\frac{\partial u}{\partial x}+b(x,y)\frac{\partial u}{\partial y}+c(x,y)u\right)=0; \tag{1}$$

this equation is stated in one of canonical forms described in [6]. In [7] one considers a problem in the triangle formed by segments of characteristics $x=\mathrm{const}$ and $y=\mathrm{const}$.

In [8] one extends the methodology and results obtained in [7] onto analogs of (1) with three and four independent variables.

In this paper we consider the equation

$$\left(\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}+\dots+\frac{\partial}{\partial x_n}\right)Lu=0, \tag{2}$$

$$Lu\equiv D^{\widetilde{\alpha}}u+\sum_{\alpha<\widetilde{\alpha}}a_\alpha(x_1,x_2,\dots,x_n)D^\alpha u,$$

where multiindices have n components, $\widetilde{\alpha}=(1,1,\dots,1)$, the dominance relation $\alpha<\widetilde{\alpha}$ means that α is obtained from $\widetilde{\alpha}$ by dropping at least one component. Obviously, (2) is an analog of (1) in the n-dimensional space.

The goal of this work is an extension of results obtained in [8] onto Eq. (2). To this end we formalize all considerations in [8], which allows us to obtain results in the space of an arbitrary number of variables.

We call solutions to (2) regular, if all derivatives of desired functions in the considered equations are continuous. Moreover, we denote by $C^{(k_1,\dots,k_n)}$ the class of functions with continuous derivatives $\partial^{s_1+\dots+s_n}u/\partial x_1^{s_1}\dots\partial x_n^{s_n}$ for all values $s_r\leqslant k_r$, $r=\overline{1,n}$.

1. The problem and its reduction to integral equations. Assume that $G=\{(x_1,x_2,\dots,x_n): 0<x_i<x_i^1, i=\overline{1,n}\}$, and $G_i=\{(x_1,x_2,\dots,x_n): 0<x_j<x_j^1, j\neq i, x_i=0\}$ are parts of the boundary G representing parts of planes $x_i=0$. We denote parts of planes $x_1=x_2,\dots,x_1=x_n$, $x_2=x_3,\dots,$ and $x_{n-1}=x_n$ lying inside G by $M_{12},\dots,M_{1n}$, $M_{23},\dots,$ and $M_{(n-1)n}$, respectively.

Let the symbol e_i stand for the unit multiindex, $e_i=(\varepsilon_1,\dots,\varepsilon_n)$, $\varepsilon_i=1$, $\varepsilon_j=0$, $j\neq i$.

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