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THE INTERPOLATION LAGRANGE POLYNOMIALS IN THE SOBOLEV SPACES

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Introduction

It is well-known that the interpolation Lagrange polynomials find various applications in different domains of both pure and applied mathematics. Therefore, since Newton and Lagrange for these polynomials there were established both positive and negative results (see, e.g., [1]–[6] and others). In particular, it was proved that even in a rather good situation (i. e., well-chosen nodes, measure of error, classes of functions to be approximated, and so on) the interpolation polynomials by their approximation properties lose against respective polynomials of best approximation. For instance, this happens in the most applicable spaces of continuous functions $C_{2\pi}$ and $C[a,b]$ and even more in the spaces of summable by Lebesgue with the power p $(1 \le p < \infty)$ 2π-periodic functions $L_p(0,2\pi)$ and in the analogous weight space of non-periodic functions $L_{p,\rho}(a,b)$, where ρ is the weight function of the interval (a,b).

In the present article we establish rather unexpected (with regard to the above-said arguments) result; apropos, the latter comes from the necessity of solving some applied problems (see, e. g., in [7]), first of all the problems of mathematical modeling in the electrodynamics:

in the Sobolev spaces, the interpolation Lagrange polynomials with equidistant pairwise nonequivalent nodes in the periodic case and with the Chebyshev nodes in nonperiodic case (these cases are to be distinguished due to some arguments of principal character) *approximate with the same rate as the respective polynomials of best approximation.*

Unexpectedness of this result is due to that in other known to us functional spaces the interpolation Lagrange polynomials fail to possess such a property.

1. Approximation by trigonometric interpolation polynomials

Let $\tilde{C} = C_{2\pi}$ and $L_2 = L_2(0,2\pi)$ be spaces of the continuous and summable in square by Lebesgue 2π-periodic functions, respectively, with the corresponding norms:

$$\|x\|_{C_{2\pi}} = \max_s |x(s)| \equiv \|x\|_\infty, \quad x \in C_{2\pi};$$

$$\|x\|_{L_2} = \left(\frac{1}{2\pi}\int_0^{2\pi} |x(\sigma)|^2\, d\sigma\right)^{1/2} \equiv \|x\|_2, \quad x \in L_2.$$

We denote by $W_2^r = W_2^r(0,2\pi)$, where r is an arbitrary positive number, the space of functions possessing generalized derivatives $D^r x(s) = x^{(r)}(s)$ of order $r \in \mathbb{R}^+$, satisfying the condition $x^{(r)} \in L_2(0,2\pi)$; here $x^{(r)}(s)$ with $r \in \mathbb{N}$ stands for the generalized in the Sobolev sense derivative, and