SPECTRAL PROPERTIES OF THE BOUNDARY VALUE PROBLEM WITH NORMAL DERIVATIVE IN THE BOUNDARY CONDITION FOR THE MIXED TYPE EQUATIONS AND THEIR APPLICATIONS

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1. Statement of the problem and main results

Let us consider the equation

$$Lu \equiv u_{xx} + \operatorname{sgn} y \cdot u_{yy} + \lambda u = 0, \qquad (1.1)$$

where the complex parameter λ lies in a domain D which is bounded by a piecewise smooth curve Γ connecting the points $A(0,0)$, $B(0,1)$ and lying in the half-plane $y > 0$, and by the characteristics AC $(x + y = 0)$ and CB $(x - y = 1)$ of equation (1.1) for $y < 0$. Let us set $D_+ = D \cap \{y > 0\}$ and $D_- = D \cap \{y < 0\}$.

We state the following problem for equation (1.1) in the domain D.

The spectral problem (TN_λ problem).
Find values of the complex parameter λ, and the corresponding functions $u(x,y)$ such that

$$u(x,y) \in C(\overline{D}) \cap C^1(D \cup \Gamma) \cap C^2(D_+ \cup D_-), \qquad (1.2)$$

$$Lu(x,y) = 0, \quad (x,y) \in D_+ \cup D_-, \qquad (1.3)$$

$$\frac{\partial u}{\partial N} = 0, \ (x,y) \in \Gamma, \qquad (1.4)$$

$$u(x,y) = 0, \quad (x,y) \in AC, \qquad (1.5)$$

where $\frac{\partial}{\partial N}$ is the normal derivative on the boundary Γ of D_+.

Important applications of the boundary problem with normal derivative in boundary condition (the TN problem) to transonic gas dynamics were demonstrated in [1], [2].

A.W. Bitsadze investigated the TN problem for equation (1.1) with $\lambda = 0$ [3]. In [4] the TN problem for equation (1.1) with $\lambda = -1$ was studied. In ([5], Ch.II, §6) the correctness of this problem was proved for the equation

$$\operatorname{sgn} y \cdot |y|^m u_{xx} + u_{yy} = 0, \quad m > 0.$$

In this paper we find the eigenvalues of the TN_λ problem and construct the corresponding system of eigenfunctions for the special domain D. We investigate the obtained system of eigenfunctions for completeness in the domain of ellipticity, in the domain of hyperbolicity, and in the mixed domain. Then, using the eigenfunction system of the TN_λ problem, we construct solutions written in the form of series to the TN problem for the equation of mixed type with Lavrentieff-Bitsadze