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CONDITIONS OF FINITE-DIMENSIONALITY OF MONODROMY OPERATOR FOR PERIODIC SYSTEMS WITH AFTEREFFECT

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1. Introduction

A linear periodic system of differential equations with aftereffect is described as follows:

$$\frac{dx(t)}{dt} = \int_{-r}^{0} d_s\eta(t,s)x(t+s), \quad t \in R^+ = [0, +\infty). \tag{1.1}$$

A matrix function $\eta : (-\infty, +\infty) \times [-r, 0] \to R^{n\times n}$ is ω-periodic with respect to the first argument and Lebesgue-measurable on the set $[0,\omega] \times [-r, 0]$, $\eta(\cdot, 0) = 0$, $\omega > r > 0$. For almost all $t \in [0,\omega]$, a finite variation $V(t) = \operatorname{Var}_{[-r,0]} \eta(t, \cdot)$ exists and the function V is integrable on $[0,\omega]$.

Under the above conditions the system of differential equations with aftereffect (1.1) for an initial moment $t_0 = 0$ and an arbitrary initial function $\varphi \in C([-r, 0], R^n)$ has a unique solution (see [1], p. 173). For this solution we introduce the notation $x(t, \varphi)$, $t \geq -r$. Here the condition $x(t, \varphi) = \varphi(t)$ is fulfilled for $t \in [-r, 0]$. It is convenient to describe the qualitative behavior of solutions of the system of differential equations with aftereffect (1.1) in the functional space of states $C([-r, 0], R^n)$. In this case, the evolution of the functional elements x_t ($x_t(s) = x(t + s)$, $s \in [-r, 0]$), $t \geq 0$ (see [2], p. 157; [3]) of this space is described by the differential equation with an unbounded operator. We denote by $x_t(\varphi)$ ($x_t(\varphi)(s) = x(t + s, \varphi)$, $s \in [-r, 0]$), $t \geq 0$, the solution of the last equation, which corresponds to both the initial moment $t_0 = 0$ and the initial value $x_0 = \varphi$.

The monodromy operator, defined by the formula $U\varphi = x_\omega(\varphi)$, acts in the space $C([-r, 0], R^n)$ and is completely continuous (see [1], p. 228). Its domain of values belongs to the space of vector-valued functions absolutely continuous on the segment $[-\omega, 0]$, possessing derivatives essentially bounded on this segment. Using the formula for a general solution of a system of differential equations with aftereffect (see [1], p. 180), we obtain the representation of the monodromy operator

$$(U\varphi)(\vartheta) = V(\omega+\vartheta, 0)\varphi(0) + \int_{-r}^{-0} d_\beta\Big(\int_0^{\omega+\vartheta} V(\omega+\vartheta,\alpha)\eta(\alpha,\beta-\alpha)d\alpha\Big)\varphi(\beta), \quad \vartheta \in [-r, 0]. \tag{1.2}$$

Here the matrix function $V(\cdot,\cdot)$ is locally absolutely continuous with respect to the first argument t on the half-interval $[\alpha, +\infty)$ for every fixed value of the second argument $\alpha \in [0, +\infty)$, has a finite

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