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ON CONVERGENCE OF FINITE DIMENSIONAL APPROXIMATIONS OF REGULARIZED SOLUTIONS IN THE THEORY OF NONLINEAR PROBLEMS

V.P. Tanana

Let H be a Hilbert space, H_n be its finite dimensional subspace, both A and A_n be operators mapping H into itself and therewith being continuous and sequentially weakly continuous. We shall denote by $\overline{A}_n$ a restriction of the operator A_n from the space H to H_n.

In the present article we introduce the notions of a weak semi-closure and weak-strong closure of the pair A, $\{A_n\}$. It will be shown that along with A-completeness of a sequence of operators $\{\overline{A}_n\}$ these conditions upon the pair A, $\{A_{n_k}\}$ are sufficient for the convergence of finite dimensional approximations in the A.N. Tikhonov regularization method (see [1]) and they are more general than the condition of the weak closure of the pair A, $\{A_{n_k}\}$ (see [2]), which was used earlier.

1. Principal definitions

Definition 1 (see [3], p. 113). A sequence $\{A_n\}$ is said to be A-complete if for any $u_0 \in H$ a sequence $\{u_n\}$ can be found such that $u_n \to u_0$ and $A_n u_n \to A u_0$.

Definition 2 (see [2]). A pair A, $\{A_n\}$ is said to be weakly closed if the fact that $u_n \rightharpoonup \widehat{u}$, while $A_n u_n \rightharpoonup \overline{f}$, implies $A\widehat{u} = \overline{f}$.

Definition 3. A pair A, $\{A_n\}$ will be termed weakly-strongly closed if the fact $u_n \rightharpoonup \widehat{u}$, while $A_n u_n \to \overline{f}$, implies $A\widehat{u} = \overline{f}$.

Definition 4. A pair A, $\{A_n\}$ is said to be weakly semi-closed if the fact $\|u_n\| \to a$, while $A_n u_n \rightharpoonup \overline{f}$, implies the existence of an element $\overline{u} \in H$ such that $\|\overline{u}\| \le a$ and $A\overline{u} = \overline{f}$.

From Definitions 2–4 it follows that a weakly closed pair A, $\{A_n\}$ is at the same time weakly-strongly closed and weakly semi-closed.

Let us give an example which shows that the inverse assertion fails. Let $H = l_2$, and let us define the operator A via the formula

$$A\overline{u} = (|u_1|, u_2, \dots), \tag{1}$$

where $\overline{u}$ and $A\overline{u} \in l_2$. Further, for any n we define the operator A_n via the formula

$$A_n\overline{u} = \Big(\sqrt{u_1^2 + u_n^2/2}\,, u_2, \dots, u_{n-1}, \frac{u_n}{\sqrt{2}}, u_{n+1}, \dots\Big), \tag{2}$$

where $\overline{u}$ and $A_n\overline{u} \in l_2$. From (1) and (2) it follows that both the operators A and A_n are continuous and sequentially weakly continuous. Moreover, $A_n\overline{u} \to A\overline{u}$ as $n \to \infty$ for any $\overline{u} \in l_2$, therefore the sequence of operators $\{A_n\}$ is A-complete.

We denote by $\{A_{n_k}\}$ an arbitrary subsequence of the operators A_n.