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Dissipative Extensions of a Symmetric Relation Generated by a System of Integral Equations with Operator Measures

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Abstract—With the help of boundary conditions we describe dissipative and accumulative extensions of the minimal relation generated by a system of integral equations with operator measures in an infinite-dimensional case.

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1. INTRODUCTION

When studying extensions of operators generated by differential expressions, one often needs to establish boundary conditions which generate extensions with some given properties. A classical example of the solution to such a problem is the description of self-adjoint extensions of a symmetric operator generated by an ordinary differential expression. Such description was given by M. G. Krein in [1] (see also [2], P. 209, Chap. 5). The method proposed by M. G. Krein essentially uses the finite dimensionality of defect subspaces of the symmetric operator. Therefore it is difficult to apply results obtained in [1] to operators with infinite defect indices.

A significant advance in overcoming these difficulties was made by F. S. Rofe-Beketov [3], who was first to use linear relations for describing self-adjoint extensions of the minimal operator generated by a differential expression with bounded operator coefficients. The results obtained in [3] were later generalized both to the case of more general (accumulative and dissipative) extensions [4] and to the case of differential expressions with unbounded operator coefficients (see monographs [5] and [6] for the detailed bibliography). Let us also mention papers [7—12], where one systematically uses linear relations when studying differential equations and inclusions.

In this paper we consider a system of integral equations with operator measures in an infinite-dimensional case. This case essentially differs from those considered above in the fact that, generally speaking, such a system generates linear relations rather than operators. The main difference consists in the fact that the space, where these linear relations take place, has a rather complicated structure. For example, the continuity of a function does not guarantee that it belongs to this space.

In the case when all measures are absolutely continuous, a system of integral equations turns into a quasidifferential expression, where quasiderivatives are understood in the sense of paper [13]. Dissipative and accumulative extensions of the minimal operator generated by such a quasidifferential expression are described in the scalar case in [14]. An important role for the description of self-adjoint extensions of minimal operators and relations is played by the construction of the space of boundary values satisfying the “Green formula”. Here for constructing boundary values we adhere to the scheme described in [15], because the boundary values obtained in papers cited above cannot be used in the case under consideration. Note that the class of integral equations with operator measures includes equations with the Stieltjes integrals [16] and differential equations whose coefficients are generalized functions. The intensive study of such differential equations was started in [17].

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