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INVESTIGATION OF A NONLINEAR STATIONARY FILTRATION PROBLEM IN THE PRESENCE OF A POINT SOURCE

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Introduction

In this paper, we investigate a mathematical model of a nonlinear stationary filtration problem for an incompressible fluid in an arbitrary bounded domain in the presence of a point source. In particular, we consider a filtration problem with the limit displacement gradient.

Concerning the function which specifies the filtration law, we assume that it demonstrates the linear growth at infinity. If the domain has a special form, the problem is solvable (e. g., [1], [2]). In [3], for an arbitrary bounded domain, it is proved that the problem, whose filtration law is described by a function with the power growth (including the linear one) at infinity, has a generalized solution. In addition, the generalized problem is formulated in the form of an equation with an operator acting from the Sobolev space ($\overset{\circ}{W}{}_2^{(1)}(\Omega)$, in case of the linear growth, Ω is the filtration domain) to the adjoined one. Correspondingly, the case when the function describing the density of external sources defines a linear continuous functional over the Sobolev space is considered. In this paper, we use the approach proposed in [3] for the investigation of a nonlinear stationary filtration equation with the less smooth right-hand side: in a non-one-dimensional case, the delta function modeling a point source does not belong to the space adjoined to $\overset{\circ}{W}{}_2^{(1)}(\Omega)$.

1. Statement of the variational problem

Consider the stationary filtration problem for an incompressible fluid, according to the nonlinear filtration law

$$w = -g(|\nabla v|)|\nabla v|^{-1}\nabla v.$$

Here w is the field of filtration rates, v is the field of fluid pressures. The filtration takes place in the domain $\Omega \subset R^n$, $n \geq 2$, with the Lipschitz continuous boundary Γ, where the pressure is assumed to be equal to zero, in the presence of a point source. The intensity of the latter at the origin (we assume that it is an interior point of Ω) equals q. The mentioned filtration process is described by the following boundary value problem:

$$-\mathrm{div}\left(\frac{g(|\nabla v(x)|)}{|\nabla v(x)|}\nabla v(x)\right) = q\delta(x), \quad x \in \Omega, \tag{1}$$

$$v(x) = 0, \quad x \in \Gamma. \tag{2}$$

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