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ON EQUATIONS WITH PERTURBED ACCRETIVE MAPPINGS

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1. Let X be a reflexive Banach space, X^* be its conjugate, X and X^* be strictly convex, $\langle y, x\rangle$ be the value of the linear functional $y \in X^*$ on the element $x \in X$, $2^X = \{Q/Q$ is a nonempty subset from $X\}$, $U : X \to X^*$ be the dual mapping in X with certain scaling function $\mu(t)$ (see [1], p. 315).

Definition 1 ([1], p. 316). An operator $A : X \to 2^X$ is said to be *accretive* if for any $x_1 \in D(A)$, $x_2 \in D(A)$ there holds the inequality

$$\langle U(x_1 - x_2),\ y_1 - y_2\rangle \geq 0 \quad \forall y_1 \in Ax_1, \quad \forall y_2 \in Ax_2.$$

Definition 2 (see, e. g., [2]). An accretive operator $A : X \to 2^X$ is said to be *maximal accretive* if its graph is not the right part of any other accretive operator acting from X to X.

Definition 3. An operator $A : X \to 2^X$ is said to be *pseudoaccretive* if

a) the set of its values at each point $x \in X$ is a convex and closed in X set;
b) from the conditions $x_n \rightharpoonup x$, $\overline{\lim}\langle U(x_n - x),\ y_n\rangle \leq 0$, $y_n \in Ax_n$ it follows that for every element $z \in X$ there can be found an element $y(z) \in Ax$ such that $\underline{\lim}\langle U(x_n - z),\ y_n\rangle \geq \langle U(x - z),\ y\rangle$.

Definition 4. We shall say that a mapping $A : X \to 2^X$ is a mapping of *type* (A) if

1) the set Ax is nonempty, bounded, convex, and closed for any $x \in X$;
2) from the conditions $x_n \rightharpoonup x$, $y_n \rightharpoonup y$, $y_n \in Ax_n$,

$$\overline{\lim}\langle Ux_n, y_n\rangle \leq \langle Ux, y\rangle \tag{1}$$

it follows that $y \in Ax$.

Definition 5. If in Definition 4 one replaces inequality (1) to

$$\overline{\lim}\langle U(x_n - x),\ y_n\rangle \leq 0, \tag{2}$$

then the operator A will be called an operator of *type* (A$'$).

If $X = H$ is a Hilbert space, $\mu(t) \equiv t$, i. e., $U = E$ is the identity operator, then the class of accretive operators coincides with the class of monotone mappings (see, e. g., [1], p. 22), the class of pseudoaccretive ones — with the class of pseudo-monotone ones (see [3], p. 96), the class of operators of type (A) and (A$'$) — with the operator of type (M) (see [3], p. 15; [4], p. 184).

Definition 6. An operator $A : X \to 2^X$ is said to be *coercive* if $\forall y \in Ax$ we have $\langle Ux, y\rangle/\|x\| \to +\infty$ as $\|x\| \to \infty$.