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Differential Geometry of Veronese-Like Webs

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1. INTRODUCTION

The notion of a Veronese web (of codimension one) on an n-dimensional manifold X^n was introduced in the paper [7] as a natural invariant of bihamiltonian structures of corank one.

In [13], [8] and [16] the authors defined Veronese webs of any codimension r, $r \geq 1$, on an (nr)-dimensional manifold X^{nr} (see also [18], [14] and [6] for this and other generalizations of Veronese webs).

First, let us remind the definition of Veronese webs of codimension r on X^{nr} given in [14] and [16] (see also [13] and [6]).

Let X^{nr} be a differentiable manifold of dimension nr. Suppose that n distributions $X_1, \ldots, X_n$ of codimension r are given on X^{nr} by systems $\{\overline{\omega}_1^i\}, \ldots, \{\overline{\omega}_n^i\}$, $i = 1, \ldots, r$, of 1-forms which are in general not completely integrable. If the system of equations

$$\omega^i(t) = \overline{\omega}_1^i + t\overline{\omega}_2^i + \cdots + t^{n-1}\overline{\omega}_n^i = 0 \qquad (1)$$

is completely integrable for any real number t, we say that a *Veronese web of codimension* r is given on X^{nr}. We shall denote such webs by $VW_t(n, r)$. Note that the set of foliations defined by Eqs. (1) is also called a *Veronese curve* of foliations (cf. [14]).

Using our notation, we can say that the authors of [7] considered the Veronese webs $VW_t(n, 1)$, and the authors of [13], [8] and [16] (see also [14] and [6]) considered the Veronese webs $VW_t(n, r)$, $r \geq 1$.

In this paper we shall study the webs which are similar to the Veronese webs (we shall call them the Veronese-like webs) for which n distributions $X_1, \ldots, X_n$ of codimension r are given on X^{nr} by systems $\{\overline{\omega}_1^i\}, \ldots, \{\overline{\omega}_n^i\}$, $i = 1, \ldots, r$, of 1-forms which are *completely integrable*, and in addition Eq. (1) is completely integrable for any real function t.

The first feature of these new webs mentioned above means that we can extend the foliations $\{\overline{\omega}_1^i = 0\}, \ldots, \{\overline{\omega}_n^i = 0\}$ to a coordinate $(n+1)$-web $W(n+1, n, r)$. This allows us to apply the results from [2], [5], [4], [9], [10] and [11].

Now we shall give the definition of such webs.

We have n sets of completely integrable 1-forms $\{\overline{\omega}_\alpha^i\}$, $\alpha = 1, \ldots, n$, i.e.,

$$d\overline{\omega}_\alpha^i \wedge \overline{\omega}_\alpha^1 \wedge \cdots \wedge \overline{\omega}_\alpha^r = 0.$$

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